US009236887B2

(12) **United States Patent**
**Luby et al.**

(10) **Patent No.: US 9,236,887 B2**
(45) **Date of Patent: Jan. 12, 2016**

(54) **FILE DOWNLOAD AND STREAMING SYSTEM**

(75) Inventors: **Michael G. Luby**, Berkeley, CA (US); **M. Amin Shokrollahi**, Preverenges (CH); **Mark Watson**, San Francisco, CA (US)

(73) Assignee: **Digital Fountain, Inc.**, San Diego, CA (US)

( * ) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **13/408,944**

(22) Filed: **Feb. 29, 2012**

(65) **Prior Publication Data**

US 2013/0067295 A1 Mar. 14, 2013

**Related U.S. Application Data**

(63) Continuation of application No. 12/197,993, filed on Aug. 25, 2008, which is a continuation of application No. 11/125,818, filed on May 9, 2005, now Pat. No. 7,418,651.

(60) Provisional application No. 60/569,127, filed on May 7, 2004.

(51) **Int. Cl.**
***G06F 11/00*** (2006.01)
***H03M 13/00*** (2006.01)
(Continued)

(52) **U.S. Cl.**
CPC .......... ***H03M 13/29*** (2013.01); ***H03M 13/3761*** (2013.01); ***H04L 1/0041*** (2013.01);
(Continued)

(58) **Field of Classification Search**
CPC .............. H03M 13/3761; H03M 13/29; H04L 1/0041; H04L 1/0061
USPC .......................................... 714/746, 799, 810
See application file for complete search history.

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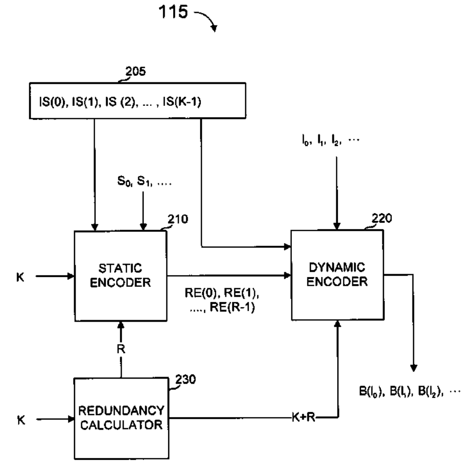
*Primary Examiner* — Shelly A Chase

(57) **ABSTRACT**

A method of encoding data operates on an ordered set of input symbols and includes generating redundant symbols from the input symbols, and includes generating output symbols from a combined set of symbols including the input symbols and the redundant symbols, wherein the number of possible output symbols is much larger than the number of the combined set of symbols, wherein at least one output symbol is generated from more than one symbol in the combined set of symbols and from less than all of the symbols in the combined set of symbols. The redundant symbols are generated from an ordered set of input symbols in a deterministic process such that a first set of static symbols calculated using a first input symbol has a low common membership with a second set of static symbols calculated using a second input symbol distinct from the first input symbol.

**27 Claims, 17 Drawing Sheets**



(51) **Int. Cl.**
***H03M 13/29*** (2006.01)
***H03M 13/37*** (2006.01)
***H04L 1/00*** (2006.01)
*H03M 13/11* (2006.01)
*H03M 13/19* (2006.01)

(52) **U.S. Cl.**
CPC ............ ***H04L1/0045*** (2013.01); ***H04L 1/0057*** (2013.01); ***H04L 1/0065*** (2013.01); *H03M 13/1102* (2013.01); *H03M 13/19* (2013.01)

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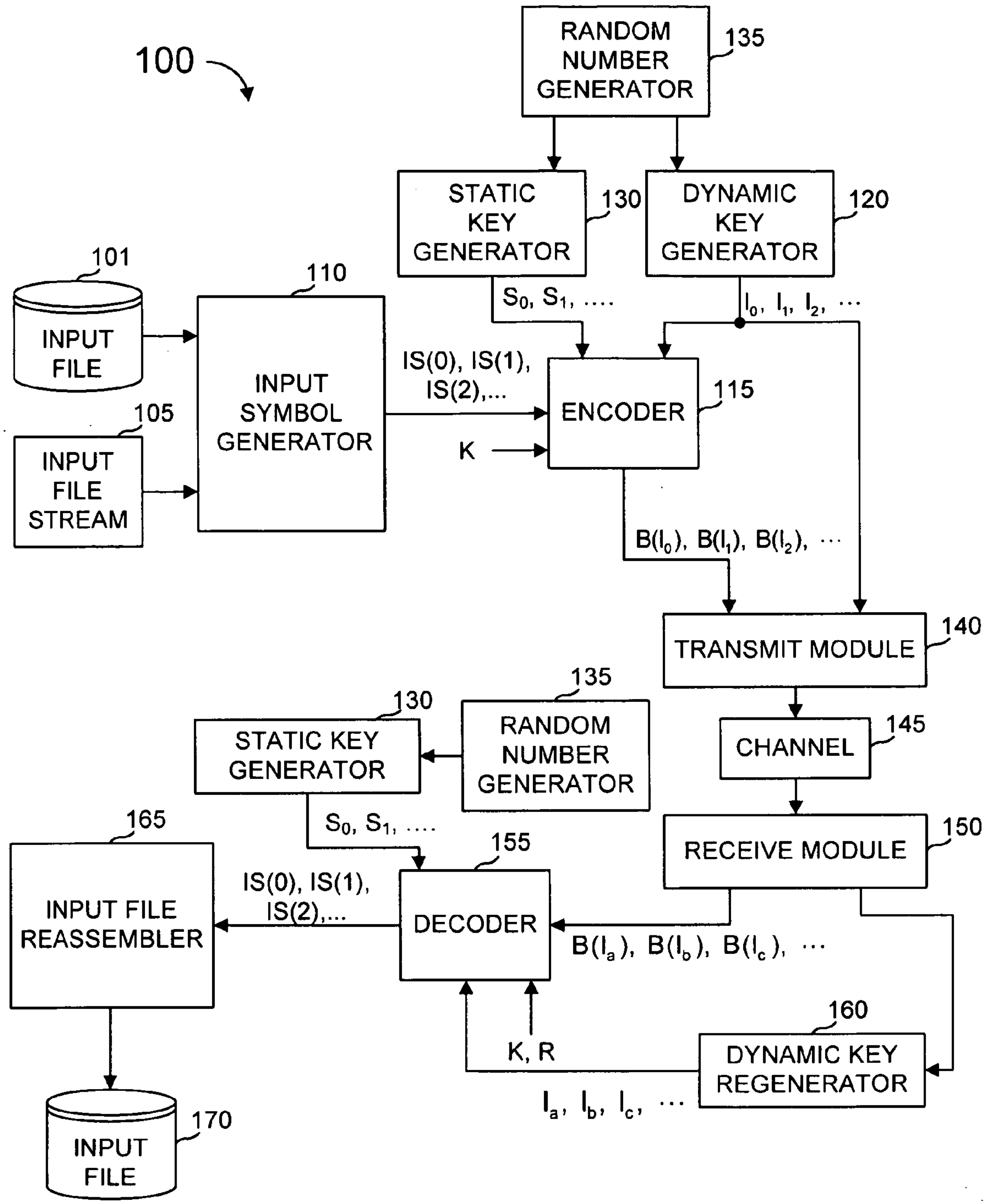
100
RANDOM NUMBER GENERATOR
135
STATIC KEY GENERATOR
130
DYNAMIC KEY GENERATOR
120
101
INPUT FILE
110
INPUT SYMBOL GENERATOR
105
INPUT FILE STREAM
S0, S1, ....
I0, I1, I2, ...
IS(0), IS(1), IS(2),...
ENCODER
115
K
B(I0), B(I1), B(I2), ...
TRANSMIT MODULE
140
135
130
STATIC KEY GENERATOR
RANDOM NUMBER GENERATOR
CHANNEL
145
165
S0, S1, ....
RECEIVE MODULE
150
155
INPUT FILE REASSEMBLER
IS(0), IS(1), IS(2),...
DECODER
B(Ia), B(Ib), B(Ic), ...
160
K, R
DYNAMIC KEY REGENERATOR
Ia, Ib, Ic, ...
INPUT FILE
170

Figure 1

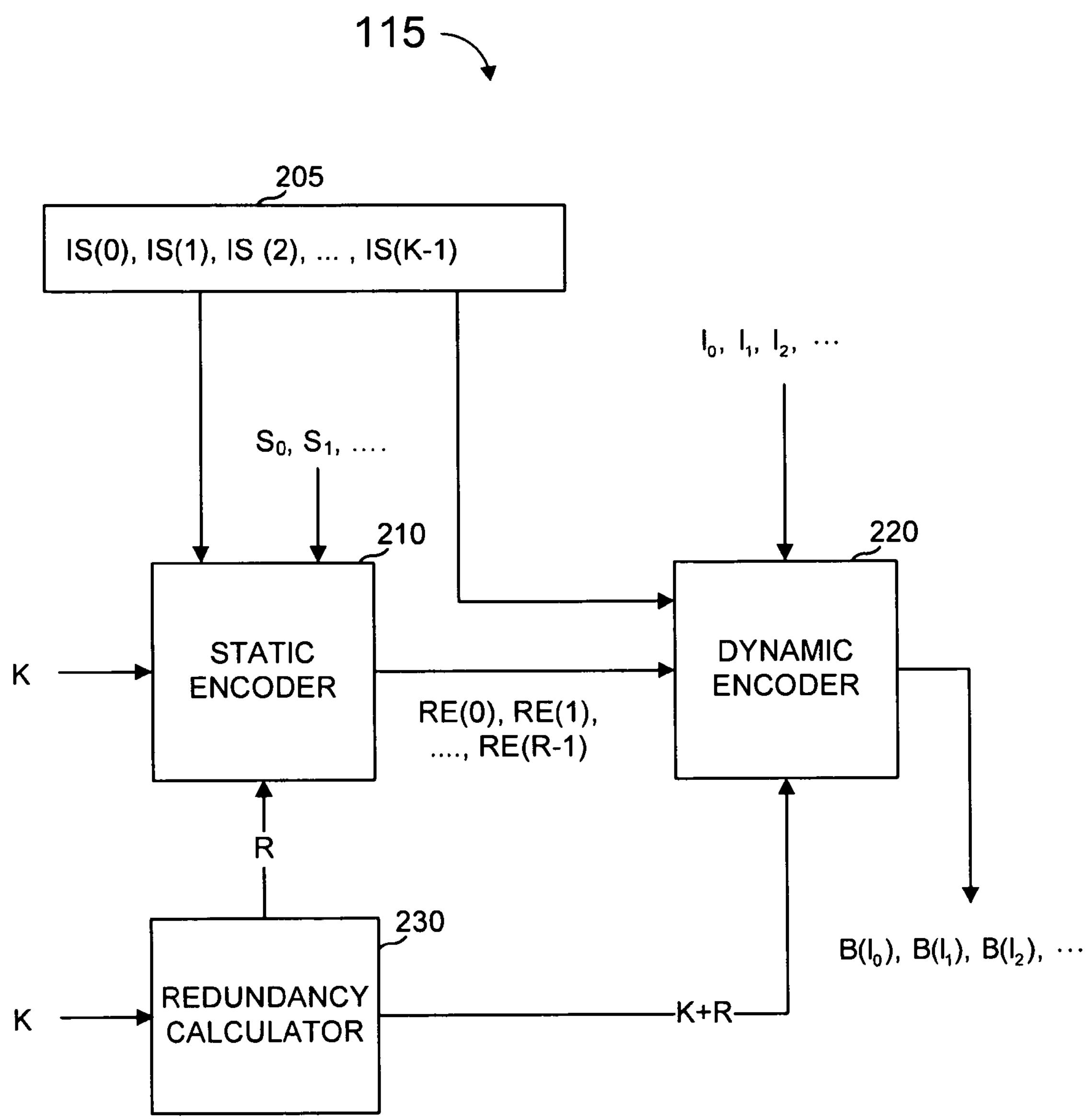
115
205
IS(0), IS(1), IS (2), ... , IS(K-1)
$I_0$, $I_1$, $I_2$, ...
$S_0$, $S_1$, ....
210
220
K
STATIC ENCODER
DYNAMIC ENCODER
RE(0), RE(1), ...., RE(R-1)
R
230
$B(I_0)$, $B(I_1)$, $B(I_2)$, ...
K
REDUNDANCY CALCULATOR
K+R

Figure 2

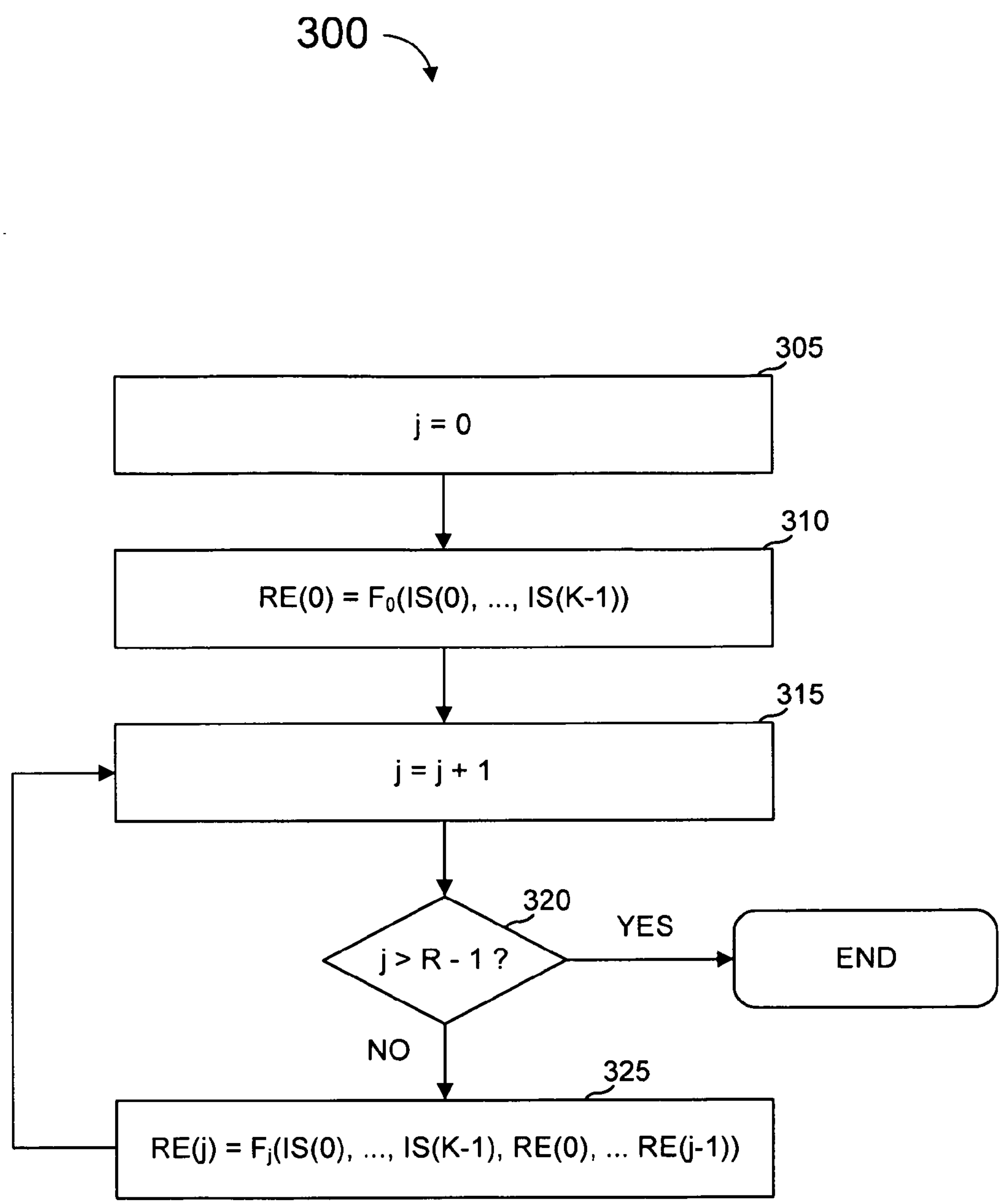
300
305
j = 0
310
RE(0) = F0(IS(0), ..., IS(K-1))
315
j = j + 1
320
j > R - 1 ?
YES
END
NO
325
RE(j) = Fj(IS(0), ..., IS(K-1), RE(0), ... RE(j-1))

Figure 3

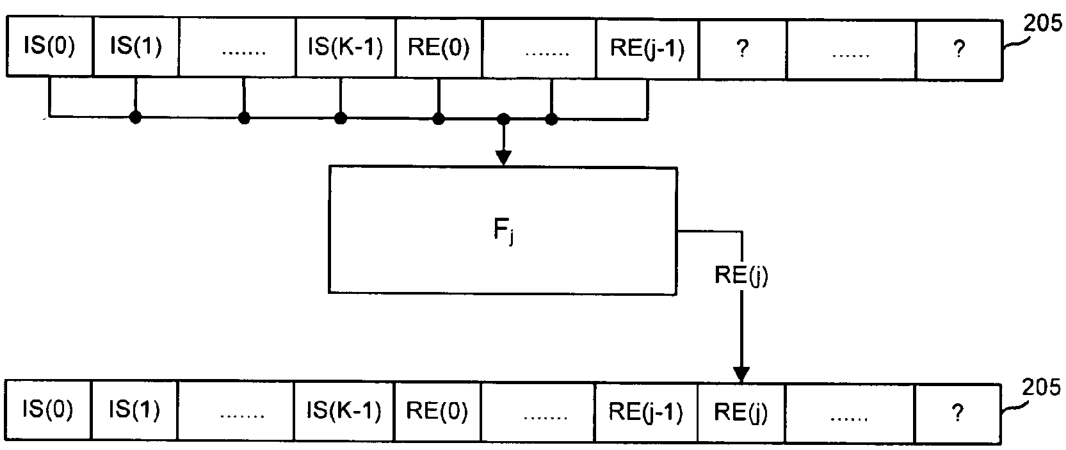


Figure 4

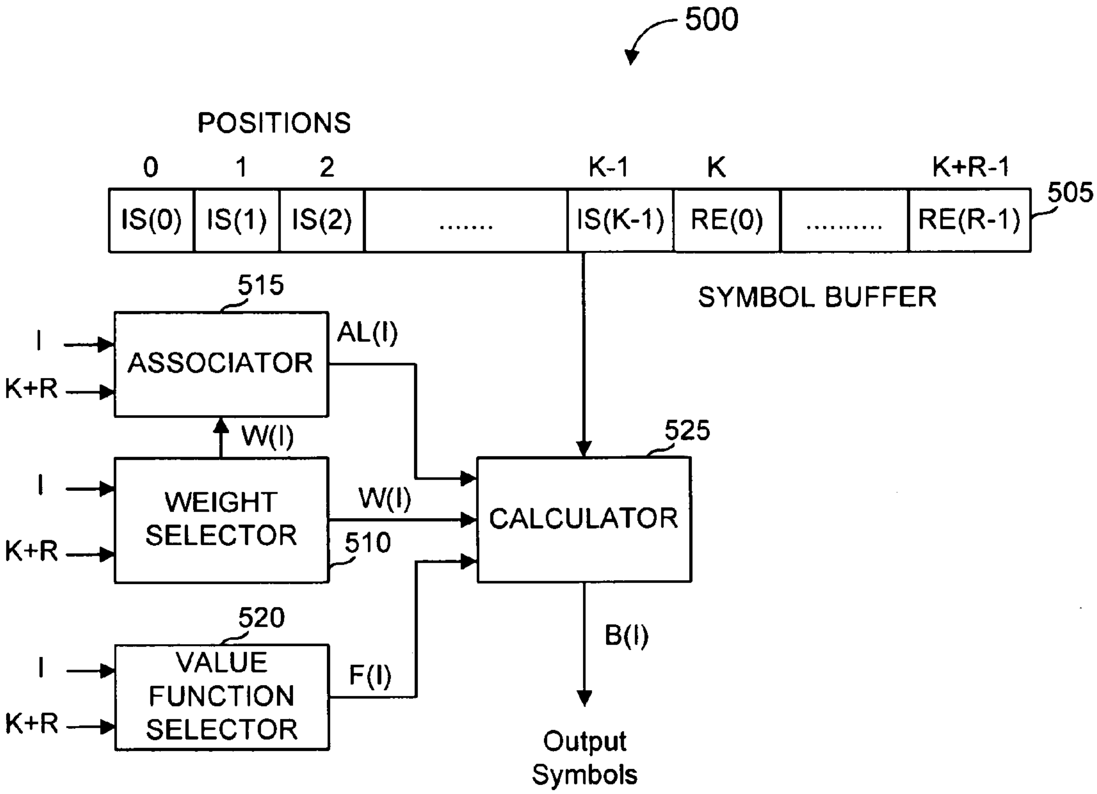


Figure 5

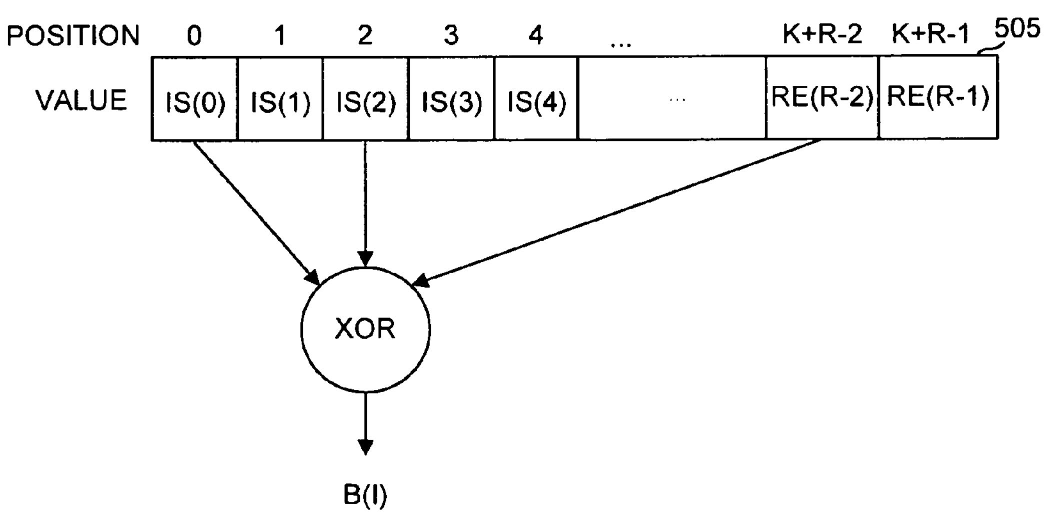


Figure 6

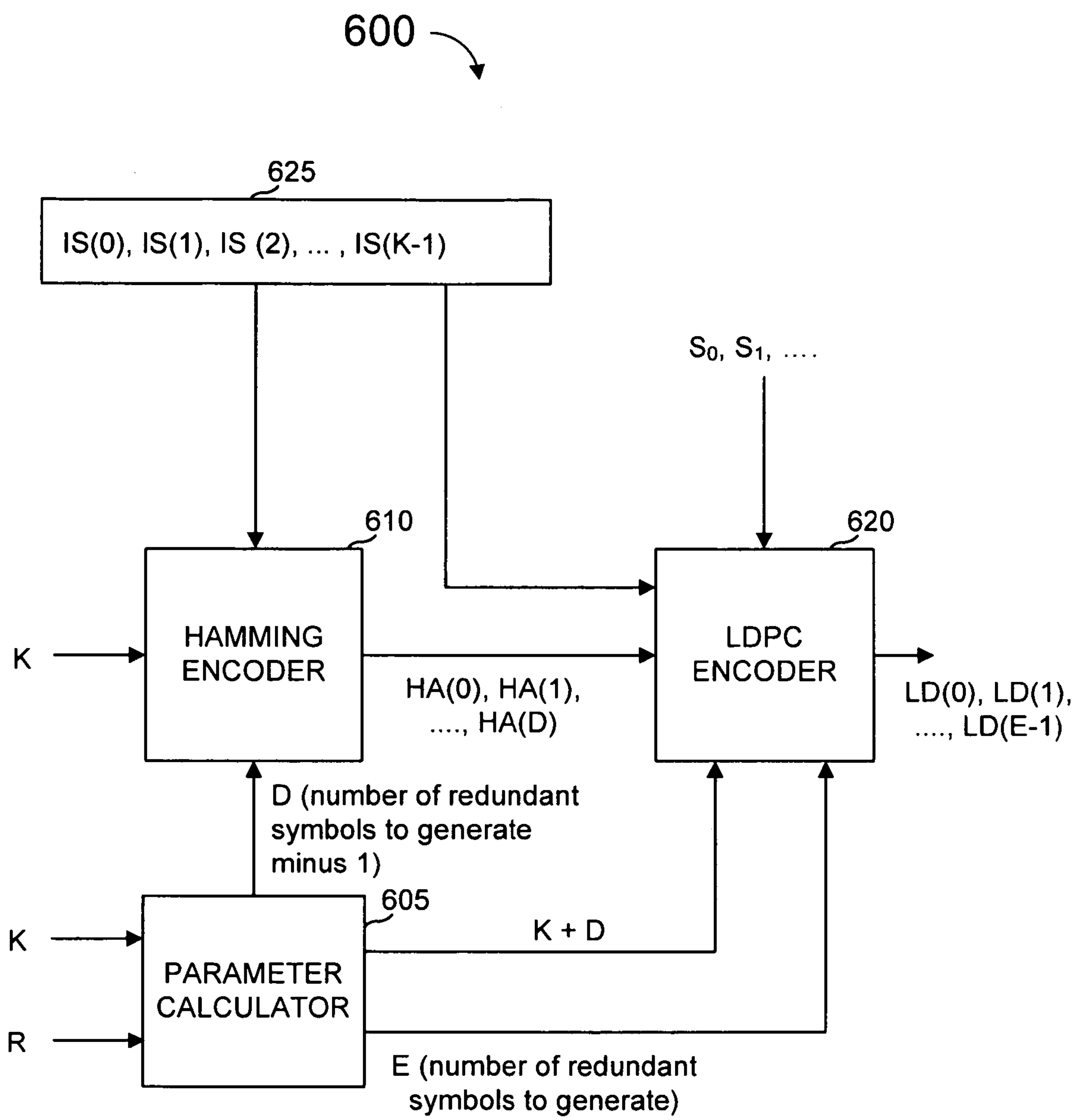
600
625
IS(0), IS(1), IS (2), ... , IS(K-1)
$S_0$, $S_1$, ....
610
620
K
HAMMING ENCODER
LDPC ENCODER
HA(0), HA(1), ...., HA(D)
LD(0), LD(1), ...., LD(E-1)
D (number of redundant symbols to generate minus 1)
605
K + D
K
R
PARAMETER CALCULATOR
E (number of redundant symbols to generate)

Figure 7

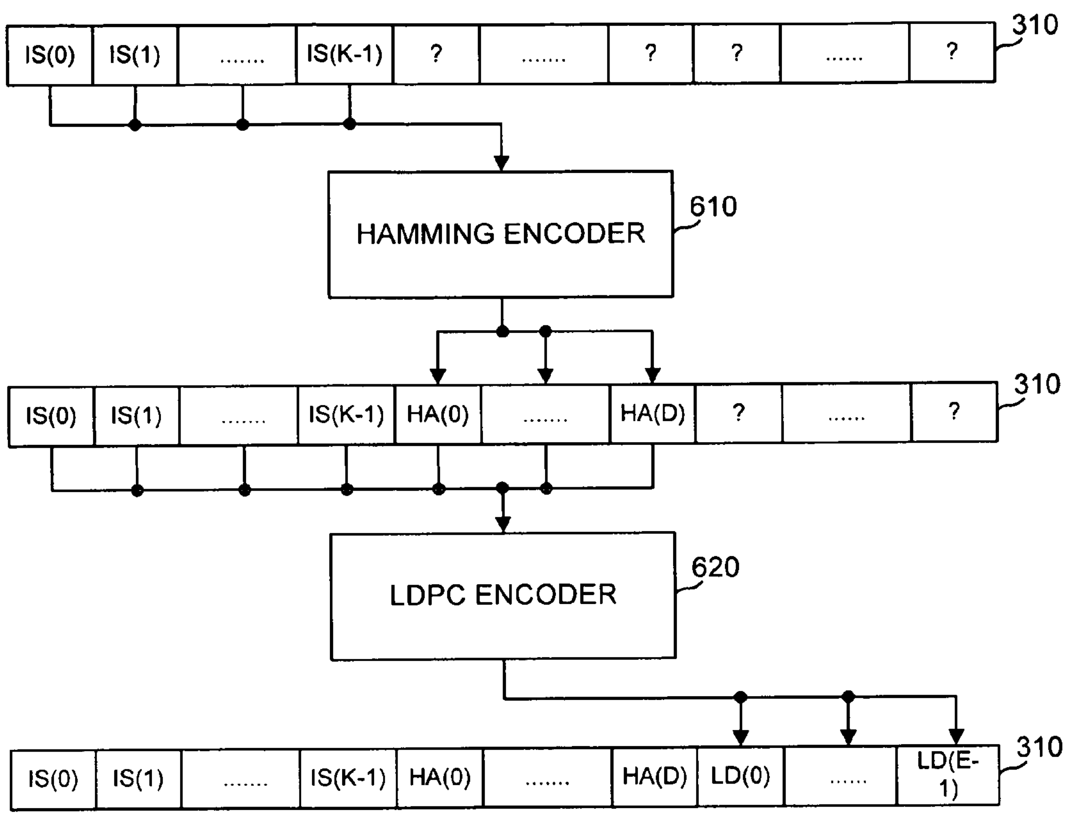
IS(0)
IS(1)
.......
IS(K-1)
?
?
?
......
?
310
HAMMING ENCODER
610
IS(0)
IS(1)
.......
IS(K-1)
HA(0)
.......
HA(D)
?
......
?
310
LDPC ENCODER
620
IS(0)
IS(1)
.......
IS(K-1)
HA(0)
.......
HA(D)
LD(0)
......
LD(E-1)
310

Figure 8

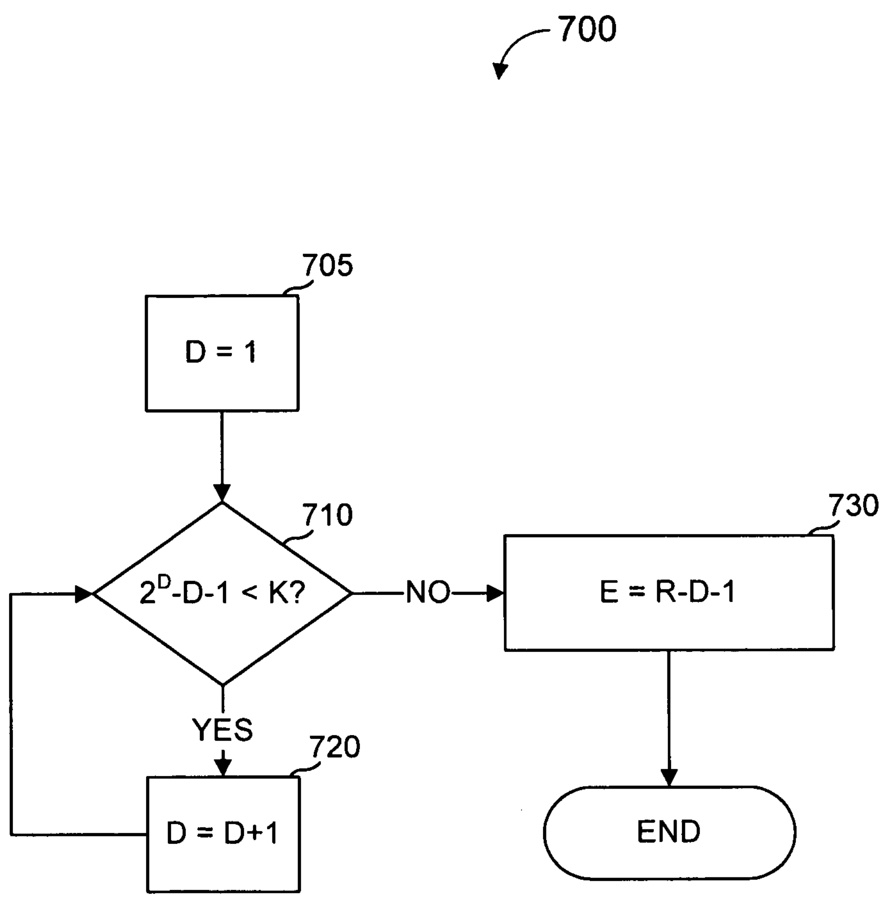


Figure 9

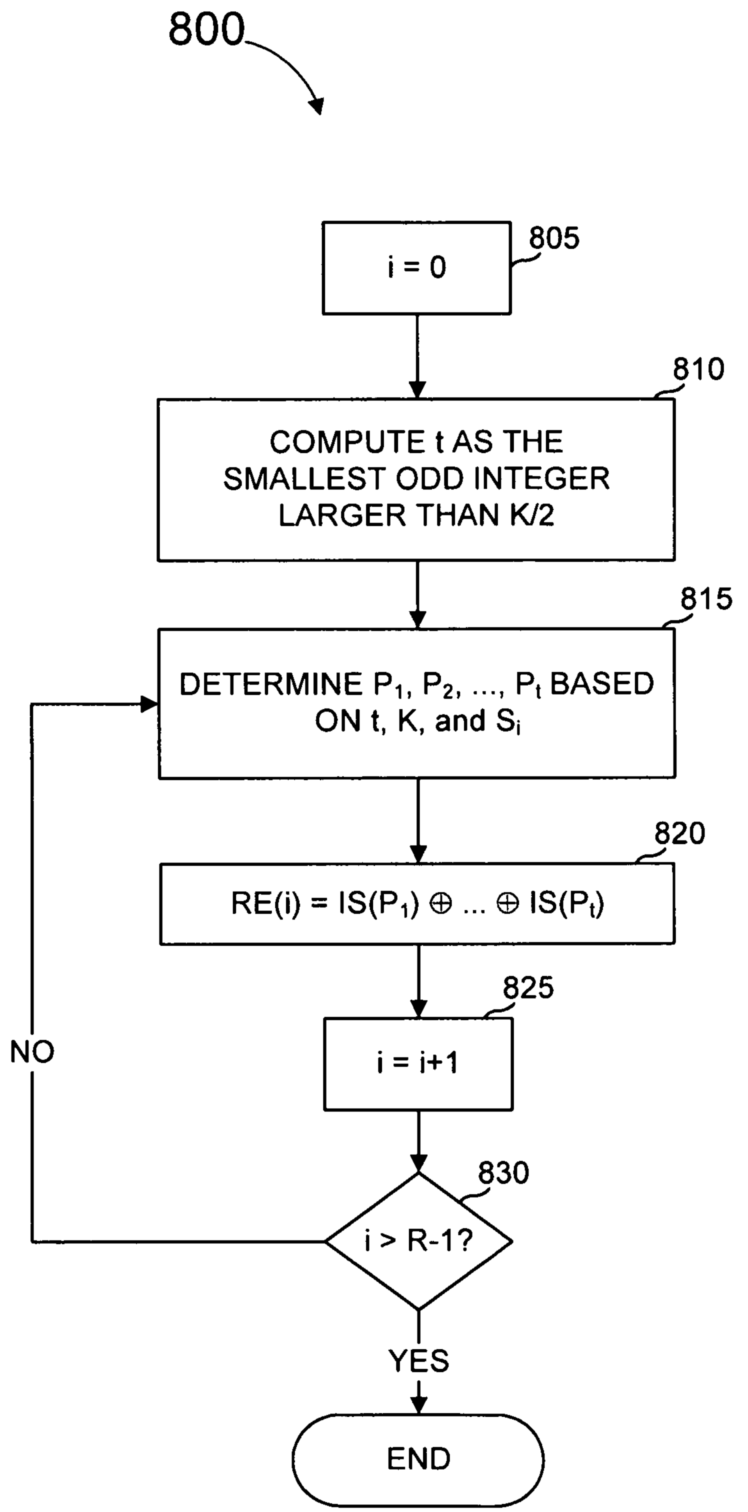
800
805
i = 0
810
COMPUTE t AS THE SMALLEST ODD INTEGER LARGER THAN K/2
815
DETERMINE P1, P2, ..., Pt BASED ON t, K, and Si
820
RE(i) = IS(P1) ⊕ ... ⊕ IS(Pt)
825
i = i+1
830
i > R-1?
NO
YES
END

Figure 10

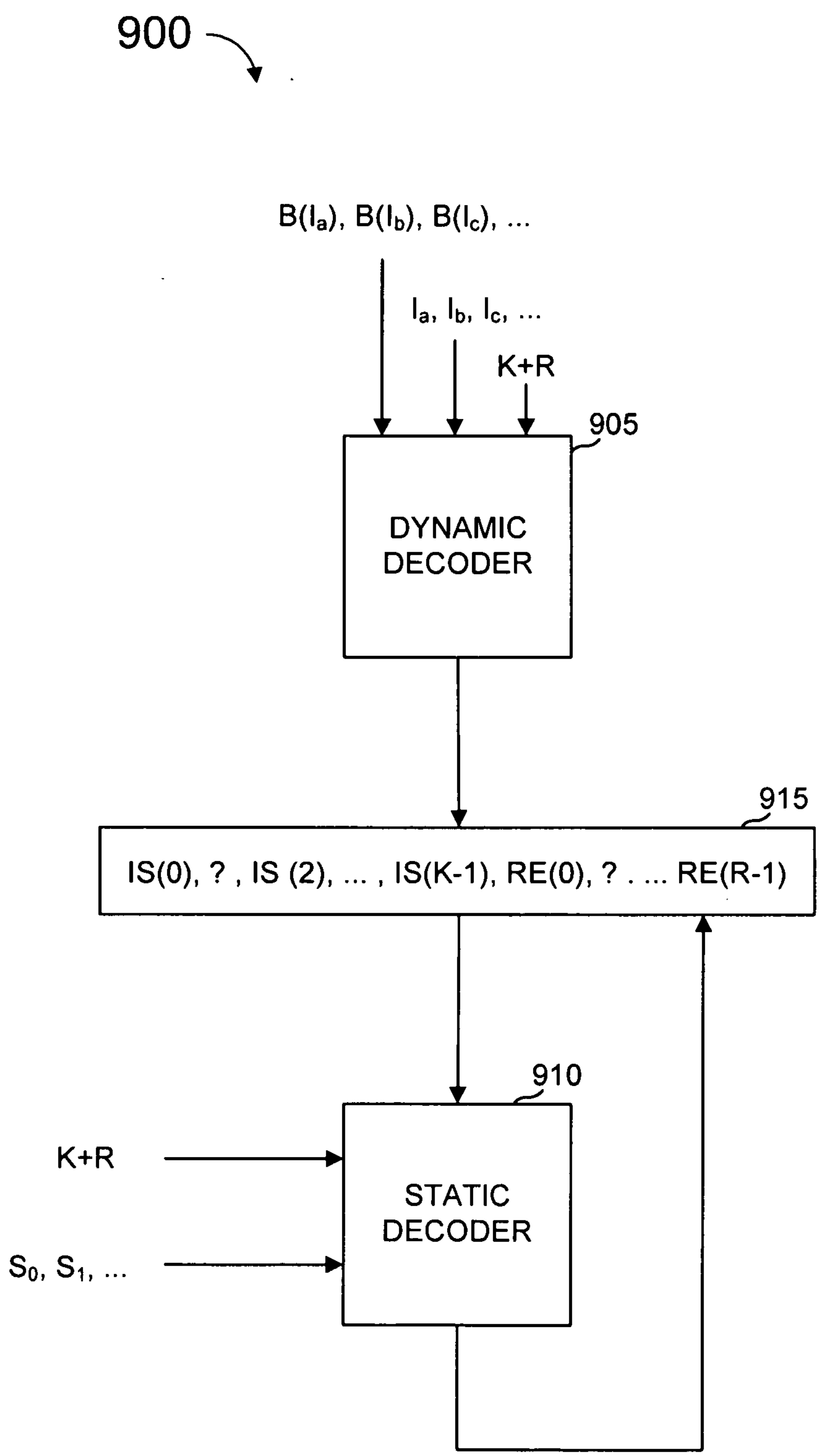
900
B($I_a$), B($I_b$), B($I_c$), ...
$I_a$, $I_b$, $I_c$, ...
K+R
905
DYNAMIC DECODER
915
IS(0), ? , IS (2), ... , IS(K-1), RE(0), ? . ... RE(R-1)
910
K+R
$S_0$, $S_1$, ...
STATIC DECODER

Figure 11

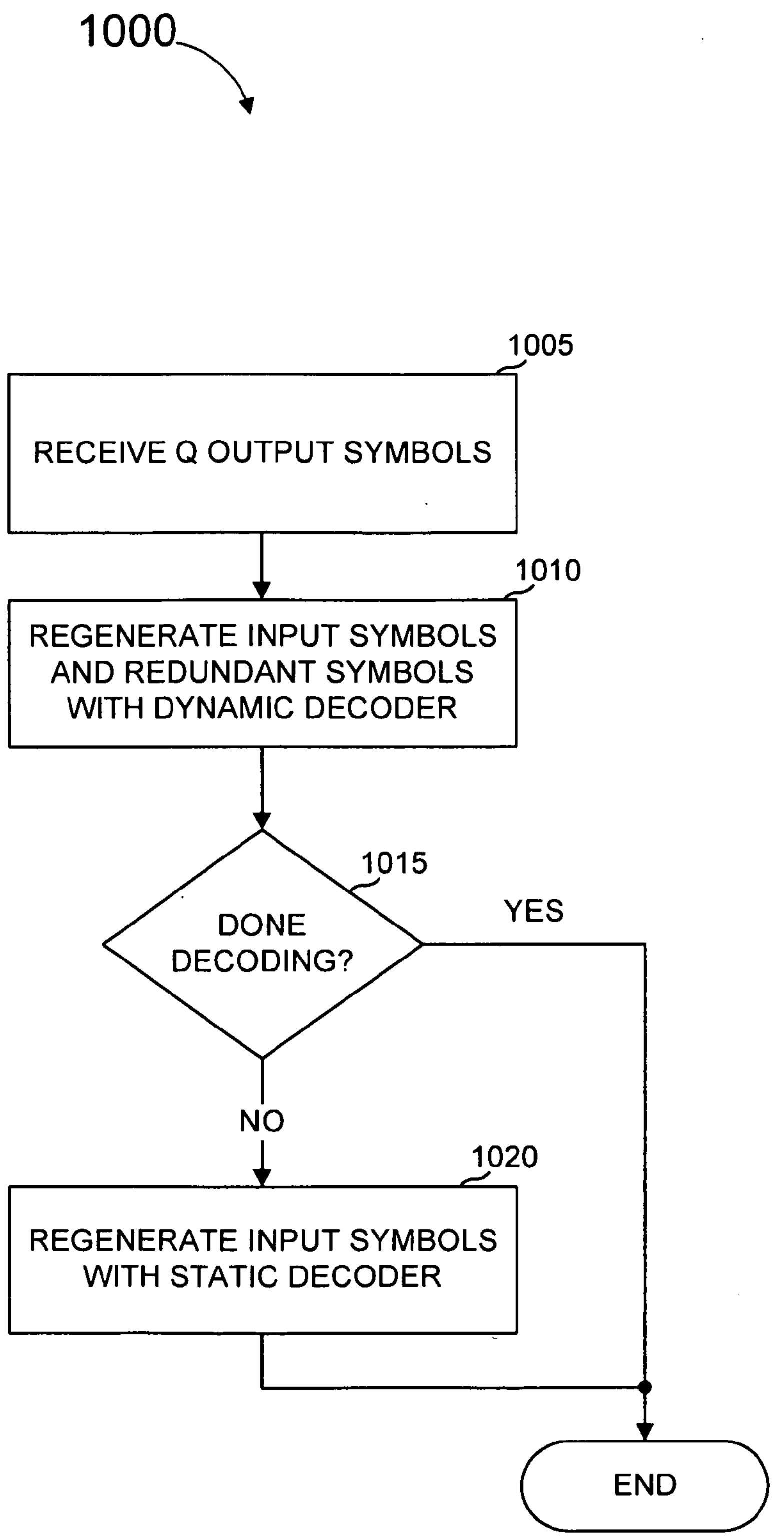
1000
1005
RECEIVE Q OUTPUT SYMBOLS
1010
REGENERATE INPUT SYMBOLS AND REDUNDANT SYMBOLS WITH DYNAMIC DECODER
1015
DONE DECODING?
YES
NO
1020
REGENERATE INPUT SYMBOLS WITH STATIC DECODER
END

Figure 12

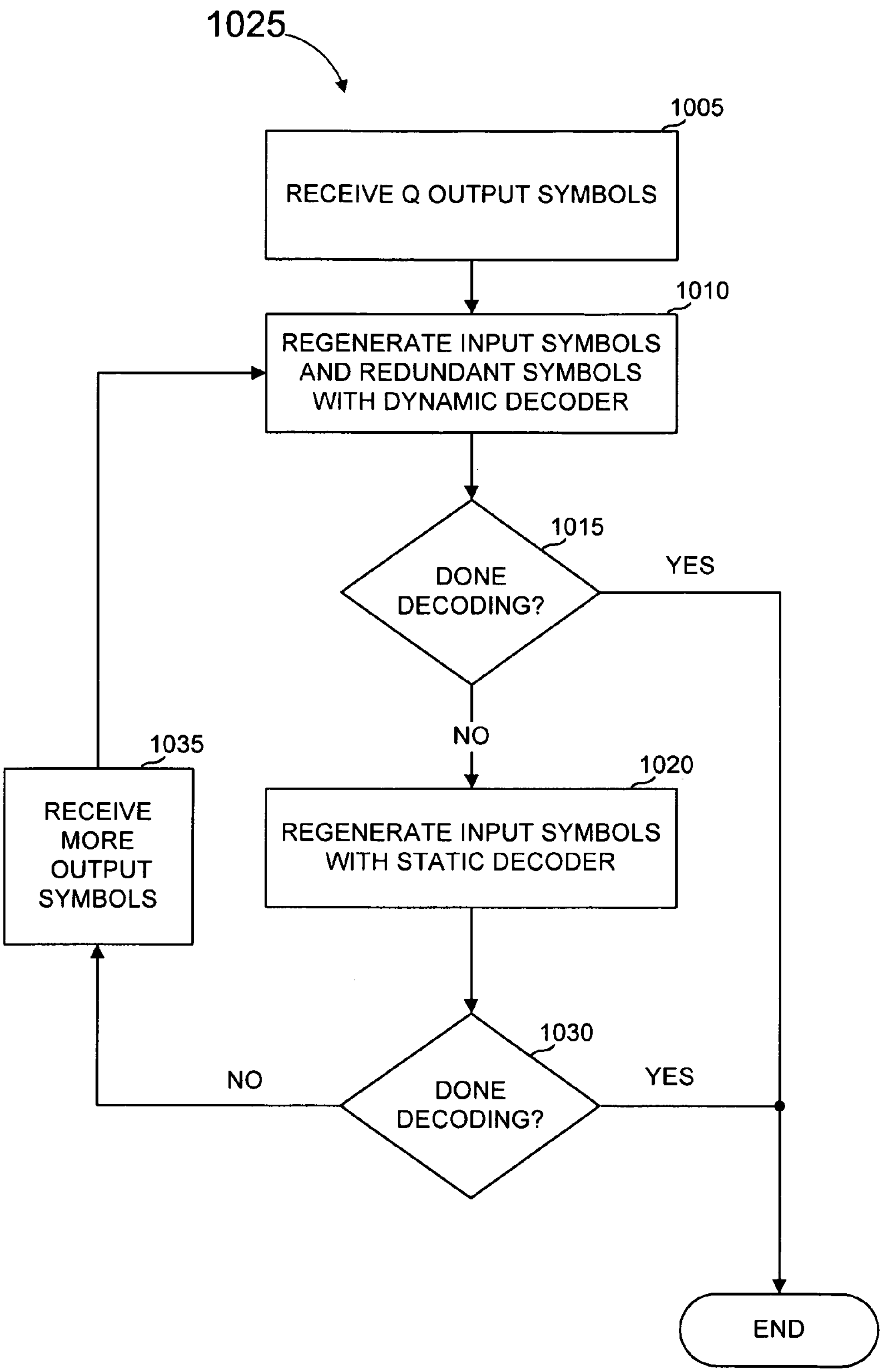
1025
1005
RECEIVE Q OUTPUT SYMBOLS
1010
REGENERATE INPUT SYMBOLS AND REDUNDANT SYMBOLS WITH DYNAMIC DECODER
1015
DONE DECODING?
YES
NO
1020
REGENERATE INPUT SYMBOLS WITH STATIC DECODER
1035
RECEIVE MORE OUTPUT SYMBOLS
1030
DONE DECODING?
NO
YES
END

Figure 13

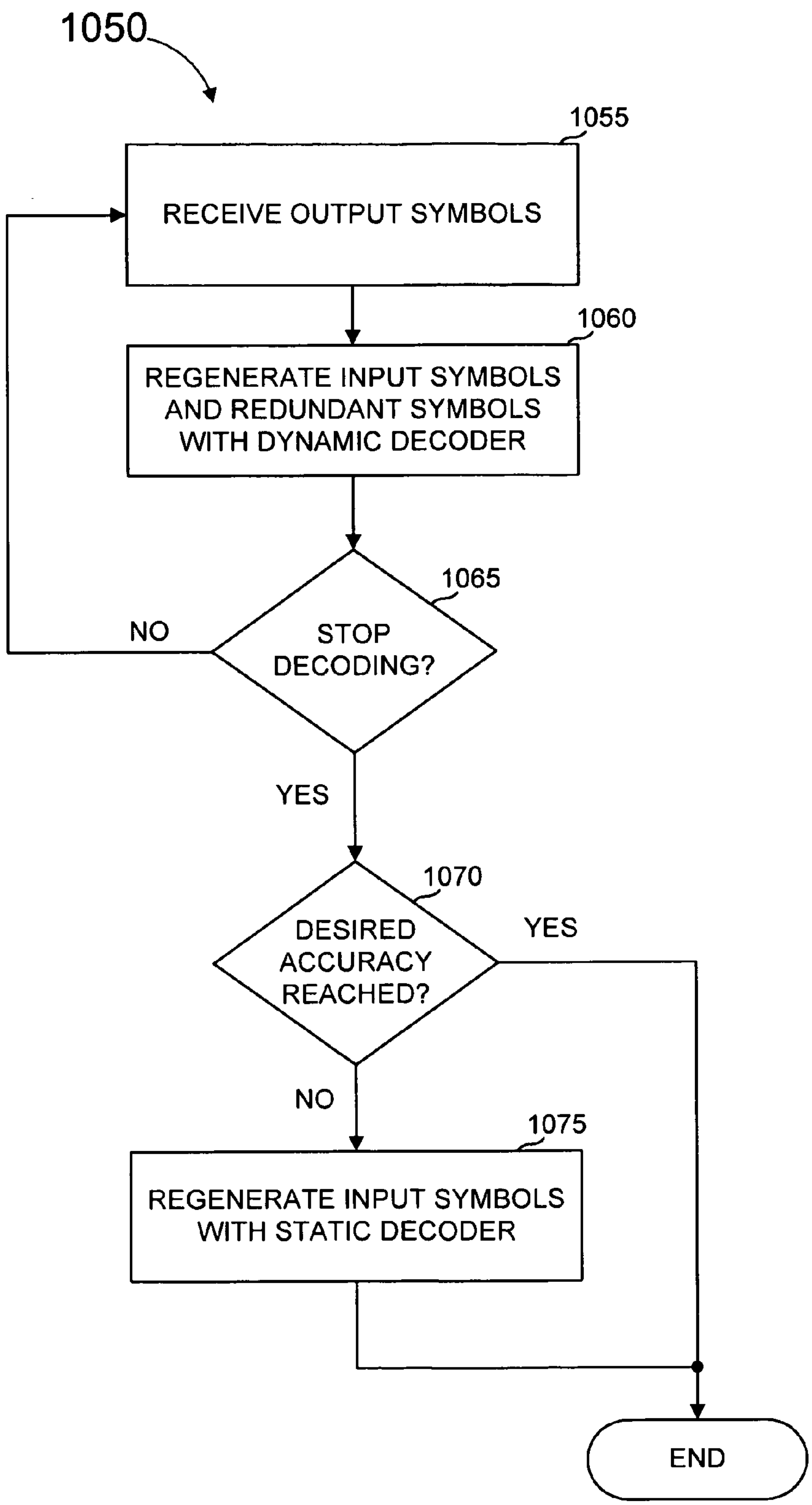
1050
1055
RECEIVE OUTPUT SYMBOLS
1060
REGENERATE INPUT SYMBOLS AND REDUNDANT SYMBOLS WITH DYNAMIC DECODER
1065
STOP DECODING?
NO
YES
1070
DESIRED ACCURACY REACHED?
YES
NO
1075
REGENERATE INPUT SYMBOLS WITH STATIC DECODER
END

Figure 14

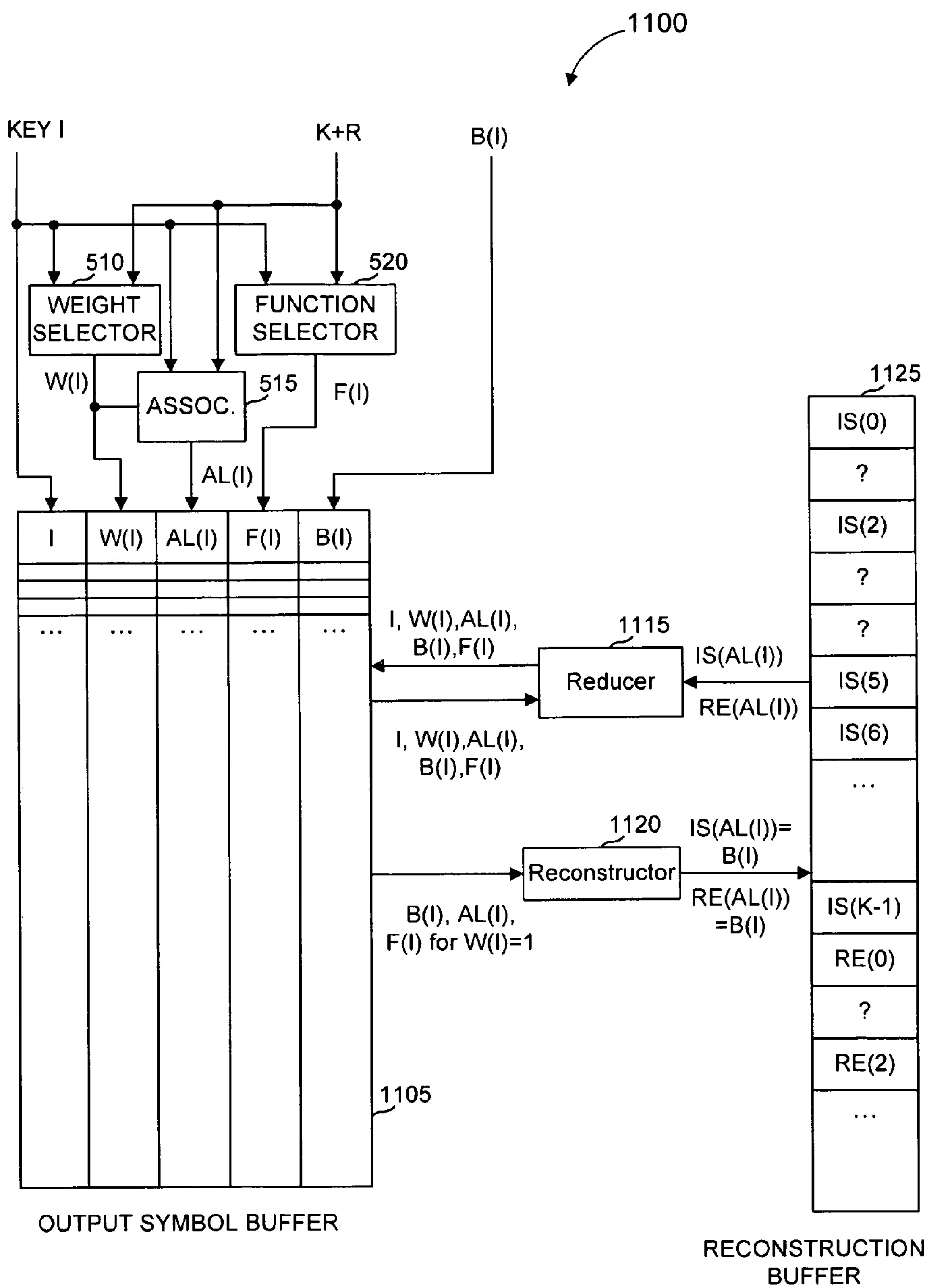
1100
KEY I
K+R
B(I)
510
520
WEIGHT SELECTOR
FUNCTION SELECTOR
W(I)
ASSOC.
515
F(I)
AL(I)
I
W(I)
AL(I)
F(I)
B(I)
I, W(I),AL(I), B(I),F(I)
1115
Reducer
IS(AL(I))
RE(AL(I))
I, W(I),AL(I), B(I),F(I)
1120
Reconstructor
IS(AL(I))= B(I)
RE(AL(I)) =B(I)
B(I), AL(I), F(I) for W(I)=1
1105
OUTPUT SYMBOL BUFFER
1125
IS(0)
?
IS(2)
?
?
IS(5)
IS(6)
IS(K-1)
RE(0)
?
RE(2)
RECONSTRUCTION BUFFER

Figure 15

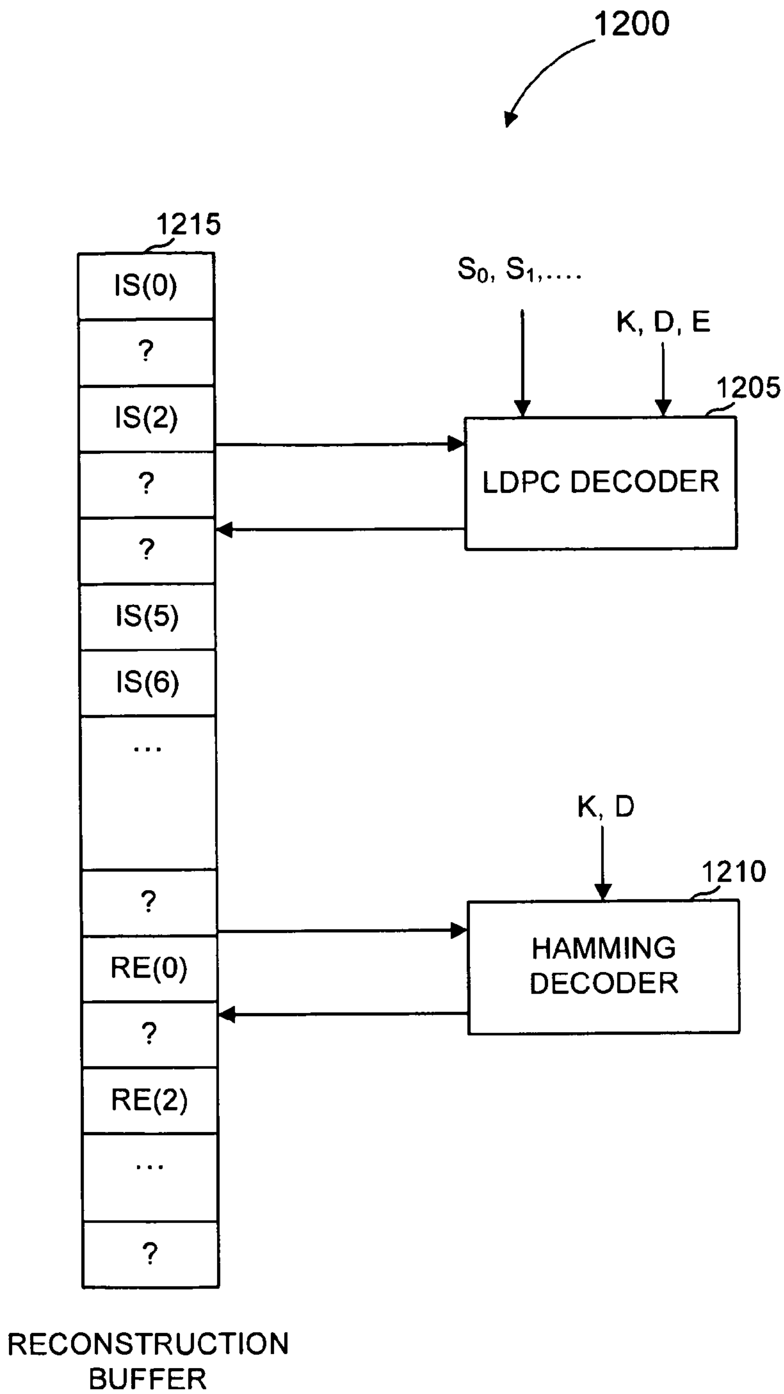
1200
1215
IS(0)
?
IS(2)
?
?
IS(5)
IS(6)
...
?
RE(0)
?
RE(2)
...
?
RECONSTRUCTION BUFFER
S0, S1,....
K, D, E
1205
LDPC DECODER
K, D
1210
HAMMING DECODER

Figure 16

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | … | … | … | … | $K$-1 |
| $K$ | $K$+1 | $K$+2 | … | … | … | … | $2\cdot K$-1 |
| $2\cdot K$ | $2\cdot K$+1 | $2\cdot K$+2 | … | … | … | … | $3\cdot K$-1 |
| … | … | … | … | … | … | … | |
| $(N\text{-}1)\cdot K$ | … | … | … | … | … | … | $N\cdot K$-1 |

Fig. 17

| e size $F$ | $G$ | Symbol size $T$ | $G$*$T$ | $K_t$ | Source blocks $Z$ | Sub-blocks $N$ | $K_L$ | $K_S$ | $T_L\cdot A$ | $T_S$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 100 KB | 6 | 84 | 504 | 1,220 | 1 | 1 | 1,220 | 1,220 | N/A | N/ |
| 100 KB | 8 | 64 | 512 | 1,600 | 1 | 1 | 1,600 | 1,600 | N/A | N/ |
| 300 KB | 2 | 256 | 512 | 1,200 | 1 | 2 | 1,200 | 1,200 | 128 | 12 |
| 000 KB | 1 | 512 | 512 | 2,000 | 1 | 5 | 2,000 | 2,000 | 104 | 10 |
| 000 KB | 1 | 512 | 512 | 6,000 | 1 | 12 | 6,000 | 6,000 | 44 | 4 |
| ,000 KB | 1 | 512 | 512 | 20,000 | 3 | 14 | 6,666 | 6,667 | 40 | 3 |

Fig. 18

| Max source block size $B$ | $G$ | Symbol size $T$ | $G\cdot T$ |
|---|---|---|---|
| 40 KB | 10 | 48 | 480 |
| 160 KB | 4 | 128 | 512 |
| 40 KB | 1 | 512 | 512 |

Fig. 19

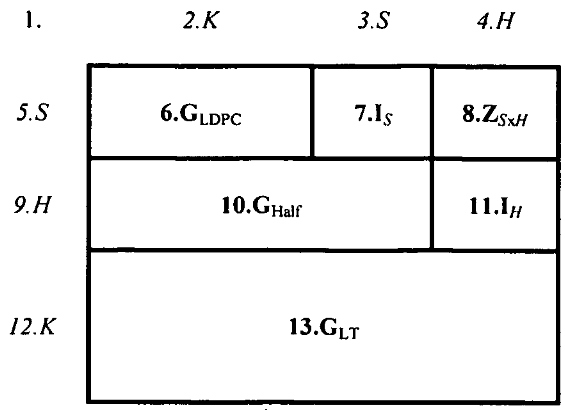

Fig. 20

| Index *j* | *f*[*j*] | *d*[*j*] |
|---|---|---|
| 0 | 0 | -- |
| 1 | 10241 | 1 |
| 2 | 491582 | 2 |
| 3 | 712794 | 3 |
| 4 | 831695 | 4 |
| 5 | 948446 | 10 |
| 6 | 1032189 | 11 |
| 7 | 1048576 | 40 |

Fig. 21

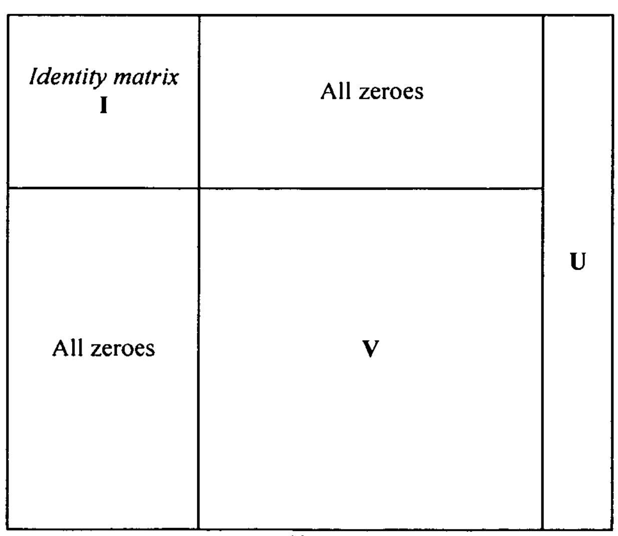

Fig. 22

# FILE DOWNLOAD AND STREAMING SYSTEM

## CROSS-REFERENCE TO RELATED APPLICATIONS

This application is a continuation of U.S. patent application Ser. No. 12/197,993, filed Aug. 25, 2008 entitled "FILE DOWNLOAD AND STREAMING SYSTEM," which is a continuation of U.S. patent application Ser. No. 11/125,818, filed May 9, 2005 entitled "FILE DOWNLOAD AND STREAMING SYSTEM," which claims priority to U.S. Provisional Patent Application No. 60/569,127, filed May 7, 2004 entitled "FILE DOWNLOAD AND STREAMING SYSTEM," each of which is hereby incorporated by reference, as if set forth in full in this document, for all purposes.

## REFERENCE TO A COMPUTER PROGRAM LISTING APPENDIX

A listing of tables, formatted as a computer program listing appendix is submitted on two duplicate compact discs ("CDs") and includes Appendices A, B.1 and B.2 as referred to herein. The computer program listing appendix is hereby incorporated by reference herein.

## FIELD OF THE INVENTION

The present invention relates to encoding and decoding data in communications systems and more specifically to communication systems that encode and decode data to account for errors and gaps in communicated data. In embodiments, data is transmitted over broadcast and/or multicast wireless networks to receivers.

## BACKGROUND OF THE INVENTION

Transmission of files and streams between a sender and a recipient over a communications channel has been the subject of much literature. Preferably, a recipient desires to receive an exact copy of data transmitted over a channel by a sender with some level of certainty. Where the channel does not have perfect fidelity (which covers most all physically realizable systems), one concern is how to deal with data lost or garbled in transmission. Lost data (erasures) are often easier to deal with than corrupted data (errors) because the recipient cannot always tell when corrupted data is data received in error. Many error-correcting codes have been developed to correct for erasures and/or for errors. Typically, the particular code used is chosen based on some information about the infidelities of the channel through which the data is being transmitted and the nature of the data being transmitted. For example, where the channel is known to have long periods of infidelity, a burst error code might be best suited for that application. Where only short, infrequent errors are expected a simple parity code might be best.

Data transmission is straightforward when a transmitter and a receiver have all of the computing power and electrical power needed for communications and the channel between the transmitter and receiver is clean enough to allow for relatively error-free communications. The problem of data transmission becomes more difficult when the channel is in an adverse environment or the transmitter and/or receiver has limited capability.

One solution is the use of forward error correcting (FEC) techniques, wherein data is coded at the transmitter such that a receiver can recover from transmission erasures and errors. Where feasible, a reverse channel from the receiver to the transmitter allows for the receiver to communicate about errors to the transmitter, which can then adjust its transmission process accordingly. Often, however, a reverse channel is not available or feasible. For example, where the transmitter is transmitting to a large number of receivers, the transmitter might not be able to handle reverse channels from all those receivers. As a result, communication protocols often need to be designed without a reverse channel and, as such, the transmitter may have to deal with widely varying channel conditions without a full view of those channel conditions.

The problem of data transmission between transmitters and receivers is made more difficult when the receivers need to be low-power, small devices that might be portable or mobile and need to receive data at high bandwidths. For example, a wireless network might be set up to deliver files or streams from a stationary transmitter to a large or indeterminate number of portable or mobile receivers either as a broadcast or multicast where the receivers are constrained in their computing power, memory size, available electrical power, antenna size, device size and other design constraints.

In such a system, considerations to be addressed include having little or no reverse channel, limited memory, limited computing cycles, mobility and timing. Preferably, the design should minimize the amount of transmission time needed to deliver data to potentially a large population of receivers, where individual receivers and might be turned on and off at unpredictable times, move in and out of range, incur losses due to link errors, cell changes, congestion in cells forcing lower priority file or stream packets to be temporarily dropped, etc.

In the case of a packet protocol used for data transport, a file, stream or other block of data to be transmitted over a packet network is partitioned into equal size input symbols and input symbols are placed into consecutive packets. The "size" of an input symbol can be measured in bits, whether or not the input symbol is actually broken into a bit stream, where an input symbol has a size of M bits when the input symbol is selected from an alphabet of $2^M$ symbols. In such a packet-based communication system, a packet oriented coding scheme might be suitable. A file transmission is called reliable if it allows the intended recipient to recover an exact copy of the original file even in the face of erasures in the network. A stream transmission is called reliable if it allows the intended recipient to recover an exact copy of each part of the stream in a timely manner even in the face of erasures in the network. Both file transmission and stream transmission can also be somewhat reliable, in the sense that some parts of the file or stream are not recoverable or for streaming if some parts of the stream are not recoverable in a timely fashion. Packet loss often occurs because sporadic congestion causes the buffering mechanism in a router to reach its capacity, forcing it to drop incoming packets. Protection against erasures during transport has been the subject of much study.

It is known to use chain reaction codes to allow for generation of an arbitrary number of output symbols from the input symbols of a file or stream. This has many uses, including the generation of output symbols in an information additive way, as opposed to an information duplicative way, wherein the latter is where a receiver receives additional data that duplicates data the receiver already knows. Novel techniques for generating, using and operating chain reaction codes are shown, for example, in U.S. Pat. No. 6,307,487 entitled "Information Additive Code Generator and Decoder for Communication Systems" issued to Luby ("Luby I"), U.S. Pat. No. 6,320,520 issued to Luby et al. entitled "Information Additive Group Code Generator and Decoder for Communication Systems" (hereinafter "Luby II"), and U.S. Pat. No. 7,068,729 issued to Shokrollahi et al. entitled "Multi-Stage Code Generator and Decoder for Communication Systems" (hereinafter "Shokrollahi"). To the extent permitted, the entire disclosures of those are herein incorporated herein by reference for all purposes.

One property of the output symbols produced by a chain reaction encoder is that a receiver is able to recover the original file or block of the original stream as soon as enough output symbols have been received. Specifically, to recover the original K input symbols with a high probability, the receiver needs approximately K+A output symbols. The ratio A/K is called the "relative reception overhead." The relative reception overhead depends on the number K of input symbols, and on the reliability of the decoder. For example, in one specific embodiment, and where K is equal to 60,000, a relative reception overhead of 5% ensures that the decoder successfully decodes the input file or block of the stream with a probability of at least $1\text{-}10^{-8}$, and where K is equal to 10,000, a relative reception overhead of 15% ensures the same success probability of the decoder. In one embodiment, the relative reception overhead of chain reaction codes can be computed as (13*sqrt(K)+200)/K, where sqrt(K) is the square root of the number of input symbols K. In this embodiment the relative reception overhead of chain reaction codes tends to be larger for small values of K.

Luby I, Luby II and Shokrollahi provide teachings of systems and methods that can be employed in certain embodiments according to the present invention. It is to be understood, however, that these systems and methods are not required of the present invention, and many other variations, modifications, or alternatives can also be used.

It is also known to use multi-stage chain reaction ("MSCR") codes, such as those described in Shokrollahi and developed by Digital Fountain, Inc. under the trade name "Raptor" codes. Multi-stage chain reaction codes are used, for example, in an encoder that receives input symbols from a source file or source stream, generates intermediate symbols therefrom and encodes the intermediate symbols using chain reaction codes. More particularly, a plurality of redundant symbols are generated from an ordered set of input symbols to be transmitted. A plurality of output symbols are generated from a combined set of symbols including the input symbols and the redundant symbols, wherein the number of possible output symbols is much larger than the number of symbols in the combined set of symbols, wherein at least one output symbol is generated from more than one symbol in the combined set of symbols and from less than all of the symbols in the combined set of symbols, and such that the ordered set of input symbols can be regenerated to a desired degree of accuracy from any predetermined number, N, of the output symbols.

For some applications, other variations of codes might be more suitable or otherwise preferred.

## BRIEF SUMMARY OF THE INVENTION

According to one embodiment of the invention, a method of encoding data for transmission from a source to a destination over a communications channel is provided. The method operates on an ordered set of input symbols and includes generating a plurality of redundant symbols from the input symbols. The method also includes generating a plurality of output symbols from a combined set of symbols including the input symbols and the redundant symbols, wherein the number of possible output symbols is much larger than the number of symbols in the combined set of symbols, wherein at least one output symbol is generated from more than one symbol in the combined set of symbols and from less than all of the symbols in the combined set of symbols, and such that the ordered set of input symbols can be regenerated to a desired degree of accuracy from any predetermined number of the output symbols. The plurality of redundant symbols is generated from an ordered set of input symbols to be transmitted in a deterministic process such that a first set of static symbols calculated using a first input symbol has a low common membership with a second set of static symbols calculated using a second input symbol distinct from the first input symbol.

According to still another embodiment of the invention, a system for receiving data transmitted from a source over a communications channel is provided using similar techniques. The system comprises a receive module coupled to a communications channel for receiving output symbols transmitted over the communications channel, wherein each output symbol is generated from at least one symbol in a combined set of input symbols and redundant symbols, wherein at least one output symbol is generated from more than one symbol in the combined set and less than all of the symbols in the combined set, wherein the number of possible output symbols is much larger than the number of symbols in the combined set, wherein the input symbols are from an ordered set of input symbols, wherein the redundant symbols are generated from the input symbols and wherein the plurality of redundant symbols is generated from an ordered set of input symbols to be transmitted in a deterministic process such that a first set of static symbols calculated using a first input symbol has a low common membership with a second set of static symbols calculated using a second input symbol distinct from the first input symbol.

According to yet another embodiment of the invention, a computer data signal embodied in a carrier wave is provided.

Numerous benefits are achieved by way of the present invention. For example, in a specific embodiment, the computational expense of encoding data for transmission over a channel is reduced. In another specific embodiment, the computational expense of decoding such data is reduced. Depending upon the embodiment, one or more of these benefits may be achieved. These and other benefits are provided in more detail throughout the present specification and more particularly below.

A further understanding of the nature and the advantages of the inventions disclosed herein may be realized by reference to the remaining portions of the specification and the attached drawings.

## BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** is a block diagram of a communications system according to one embodiment of the present invention.

FIG. **2** is a block diagram an encoder according to one embodiment of the present invention.

FIG. **3** is a simplified block diagram of a method of generating redundant symbols according to one embodiment of the present invention.

FIG. **4** is a simplified block diagram of the basic operation of a static encoder according to one embodiment of the present invention.

FIG. **5** is a simplified block diagram of a dynamic encoder according to one embodiment of the present invention.

FIG. **6** is a simplified block diagram of a basic operation of a dynamic encoder according to one embodiment of the present invention.

FIG. **7** is a simplified block diagram of a static encoder according to one embodiment of the present invention.

FIG. **8** is a simplified block diagram of the basic operation a static encoder according to one embodiment of the present invention.

FIG. **9** is a simplified diagram of a method for calculating encoding parameters according to one specific embodiment of a static encoder.

FIG. **10** is a simplified flow diagram of a static encoder according to another embodiment of the present invention.

FIG. **11** is a simplified block diagram of a decoder according to one embodiment of the present invention.

FIG. **12** is a simplified flow diagram of an operation of a decoder according to one embodiment of the present invention.

FIG. **13** is a simplified flow diagram of an operation of a decoder according to another embodiment of the present invention.

FIG. **14** is a simplified flow diagram of an operation of a decoder according to yet another embodiment of the present invention.

FIG. **15** is a simplified block diagram of a dynamic decoder according to one embodiment of the present invention.

FIG. **16** is a simplified block diagram of a static decoder according to one embodiment of the present invention.

FIG. **17** illustrates source symbol from sub-symbol mappings.

FIG. **18** illustrates possible settings of file download parameters for various file sizes.

FIG. **19** illustrates possible settings of streaming parameters for various source block sizes.

FIG. **20** illustrates a form of a matrix that represents a relationship between source and intermediate symbols.

FIG. **21** illustrates a degree distribution for the degree generator.

FIG. **22** illustrates a form of the matrix A that can be used for decoding.

A listing of tables, formatted as a computer program listing appendix is submitted on two duplicate compact discs ("CDs") and includes Appendices A, B.1 and B.2 as described in this paragraph and are hereby incorporated by reference herein. Appendix A provides an example of a table of Systematic Indices J(K). For each value of K, the systematic index J(K) is designed to have the property that the set of source symbol triples (d[0], a[0], b[0]), . . . , (d[L−1], a[L−1], b[L−1]) are such that the L intermediate symbols are uniquely defined, i.e., the matrix A in Section B.5.2.4.2 has full rank and is therefore invertible. Appendix A provides the list of the systematic indices for values of K between 4 and 8192 inclusive. The order of the values begins with the index for K=4 and ends with index for K=8192. Appendix B.1 provides an example of table $V_0$. These values represent an example set of values for Table $V_0$ described in Section B.5.4.1. Each entry is a 32-bit integer in decimal representation. The order of the values is from the first line to the last line. Appendix B.2 provides an example of table $V_1$. These values represent an example set of values for Table $V_1$ described in Section B.5.4.1. Each entry is a 32-bit integer in decimal representation. The order of the values is from the first line to the last line.

## DETAILED DESCRIPTION OF SPECIFIC EMBODIMENTS

In the specific embodiments described herein, a coding scheme denoted as "multi-stage coding" is described, embodiments of which are provided in Shokrollahi.

Multi-stage encoding, as described herein, encodes the data in a plurality of stages. Typically, but not always, a first stage adds a predetermined amount of redundancy to the data. A second stage then uses a chain reaction code, or the like, to produce output symbols from the original data and the redundant symbols computed by the first stage of the encoding. In one specific embodiment of the present invention, the received data is first decoded using a chain reaction decoding process. If that process is not successful in recovering the original data completely, a second decoding step can be applied.

In embodiments of multi-stage encoding, redundant symbols are generated from the input file or block of the stream during the first stage of encoding. In these embodiments, in the second stage of encoding, output symbols are generated from the combination of the input file or block of the stream and the redundant symbols. In some of these embodiments, the output symbols can be generated as needed. In embodiments in which the second stage comprises chain reaction encoding, each output symbol can be generated without regard to how other output symbols are generated. Once generated, these output symbols can then be placed into packets and transmitted to their destination, with each packet containing one or more output symbols. Non-packetized transmission techniques can be used instead or as well.

As used herein, the term "file" refers to any data that is stored at one or more sources and is to be delivered as a unit to one or more destinations. Thus, a document, an image, and a file from a file server or computer storage device, are all examples of "files" that can be delivered. Files can be of known size (such as a one megabyte image stored on a hard disk) or can be of unknown size (such as a file taken from the output of a streaming source). Either way, the file is a sequence of input symbols, where each input symbol has a position in the file and a value.

As used herein, the term "stream" refers to any data that is stored or generated at one or more sources and is delivered at a specified rate at each point in time in the order it is generated to one or more destinations. Streams can be fixed rate or variable rate. Thus, an MPEG video stream, AMR audio stream, and a data stream used to control a remote device, are all examples of "streams" that can be delivered. The rate of the stream at each point in time can be known (such as 4 megabits per second) or unknown (such as a variable rate stream where the rate at each point in time is not known in advance). Either way, the stream is a sequence of input symbols, where each input symbol has a position in the stream and a value.

Transmission is the process of transmitting data from one or more senders to one or more recipients through a channel in order to deliver a file or stream. A sender is also sometimes referred to as the encoder. If one sender is connected to any number of recipients by a perfect channel, the received data can be an exact copy of the input file or stream, as all the data will be received correctly. Here, we assume that the channel is not perfect, which is the case for most real-world channels. Of the many channel imperfections, two imperfections of interest are data erasure and data incompleteness (which can be treated as a special case of data erasure). Data erasure occurs when the channel loses or drops data. Data incompleteness occurs when a recipient does not start receiving data until some of the data has already passed it by, the recipient stops receiving data before transmission ends, the recipient chooses to only receive a portion of the transmitted data, and/or the recipient intermittently stops and starts again receiving data. As an example of data incompleteness, a moving satellite sender might be transmitting data representing an input file or stream and start the transmission before a recipient is in range. Once the recipient is in range, data can be received until the satellite moves out of range, at which point the recipient can redirect its satellite dish (during which time it is not receiving data) to start receiving the data about the same input file or stream being transmitted by another satellite that has moved into range. As should be apparent from reading this description, data incompleteness is a special case of data erasure, since the recipient can treat the data incompleteness (and the recipient has the same problems) as if the recipient was in range the entire time, but the channel lost all the data up to the point where the recipient started receiving data. Also, as is well known in communication systems design, detectable errors can be considered equivalent to erasures by simply dropping all data blocks or symbols that have detectable errors.

In some communication systems, a recipient receives data generated by multiple senders, or by one sender using multiple connections. For example, to speed up a download, a recipient might simultaneously connect to more than one sender to transmit data concerning the same file. As another example, in a multicast transmission, multiple multicast data streams might be transmitted to allow recipients to connect to one or more of these streams to match the aggregate transmission rate with the bandwidth of the channel connecting them to the sender. In all such cases, a concern is to ensure that all transmitted data is of independent use to a recipient, i.e., that the multiple source data is not redundant among the streams, even when the transmission rates are vastly different for the different streams, and when there are arbitrary patterns of loss.

In general, a communication channel is that which connects the sender and the recipient for data transmission. The communication channel could be a real-time channel, where the channel moves data from the sender to the recipient as the channel gets the data, or the communication channel might be a storage channel that stores some or all of the data in its transit from the sender to the recipient. An example of the latter is disk storage or other storage device. In that example, a program or device that generates data can be thought of as the sender, transmitting the data to a storage device. The recipient is the program or device that reads the data from the storage device. The mechanisms that the sender uses to get the data onto the storage device, the storage device itself and the mechanisms that the recipient uses to get the data from the storage device collectively form the channel. If there is a chance that those mechanisms or the storage device can lose data, then that would be treated as data erasure in the communication channel.

When the sender and recipient are separated by a communication channel in which symbols can be erased, it is preferable not to transmit an exact copy of an input file or stream, but instead to transmit data generated from the input file or stream (which could include all or parts of the input file or stream itself) that assists with recovery of erasures. An encoder is a circuit, device, module or code segment that handles that task. One way of viewing the operation of the encoder is that the encoder generates output symbols from input symbols, where a sequence of input symbol values represent the input file or a block of the stream. Each input symbol would thus have a position, in the input file or block of the stream, and a value. A decoder is a circuit, device, module or code segment that reconstructs the input symbols from the output symbols received by the recipient. In multi-stage coding, the encoder and the decoder are further divided into sub-modules each performing a different task.

In embodiments of multi-stage coding systems, the encoder and the decoder can be further divided into sub-modules, each performing a different task. For instance, in some embodiments, the encoder comprises what is referred to herein as a static encoder and a dynamic encoder. As used herein, a "static encoder" is an encoder that generates a number of redundant symbols from a set of input symbols, wherein the number of redundant symbols is determined prior to encoding. Examples of static encoding codes include Reed-Solomon codes, Tornado codes, Hamming codes, Low Density Parity Check (LDPC) codes, etc. The term "static decoder" is used herein to refer to a decoder that can decode data that was encoded by a static encoder.

As used herein, a "dynamic encoder" is an encoder that generates output symbols from a set of input symbols, where the number of possible output symbols is orders of magnitude larger than the number of input symbols, and where the number of output symbols to be generated need not be fixed. One example of a dynamic encoder is a chain reaction encoder, such as the encoders described in Luby I and Luby II. The term "dynamic decoder" is used herein to refer to a decoder that can decode data that was encoded by a dynamic encoder.

Embodiments of multi-stage coding need not be limited to any particular type of input symbol. Typically, the values for the input symbols are selected from an alphabet of $2^M$ symbols for some positive integer M. In such cases, an input symbol can be represented by a sequence of M bits of data from the input file or stream. The value of M is often determined based on, for example, the uses of the application, the communication channel, and/or the size of the output symbols. Additionally, the size of an output symbol is often determined based on the application, the channel, and/or the size of the input symbols. In some cases, the coding process might be simplified if the output symbol values and the input symbol values were the same size (i.e., representable by the same number of bits or selected from the same alphabet). If that is the case, then the input symbol value size is limited when the output symbol value size is limited. For example, it may be desired to put output symbols in packets of limited size. If some data about a key associated with the output symbols were to be transmitted in order to recover the key at the receiver, the output symbol would preferably be small enough to accommodate, in one packet, the output symbol value and the data about the key.

As an example, if an input file is a multiple megabyte file, the input file might be broken into thousands, tens of thousands, or hundreds of thousands of input symbols with each input symbol encoding thousands, hundreds, or only few bytes. As another example, for a packet-based Internet channel, a packet with a payload of size of 1024 bytes might be appropriate (a byte is 8 bits). In this example, assuming each packet contains one output symbol and 8 bytes of auxiliary information, an output symbol size of 8128 bits ((1024−8)*8) would be appropriate. Thus, the input symbol size could be chosen as M=(1024−8)*8, or 8128 bits. As another example, some satellite systems use the MPEG packet standard, where the payload of each packet comprises 188 bytes. In that example, assuming each packet contains one output symbol and 4 bytes of auxiliary information, an output symbol size of 1472 bits ((188−4)*8), would be appropriate. Thus, the input symbol size could be chosen as M=(188−4)*8, or 1472 bits. In a general-purpose communication system using multi-stage coding, the application-specific parameters, such as the input symbol size (i.e., M, the number of bits encoded by an input symbol), might be variables set by the application.

As another example, for a stream that is sent using variable size source packets, the symbol size might be chosen to be rather small so that each source packet can be covered with an integral number of input symbols that have aggregate size at most slightly larger than the source packet.

Each output symbol has a value. In one preferred embodiment, which we consider below, each output symbol also has associated therewith an identifier called its "key." Preferably, the key of each output symbol can be easily determined by the recipient to allow the recipient to distinguish one output symbol from other output symbols. Preferably, the key of an output symbol is distinct from the keys of all other output symbols. There are various forms of keying discussed in previous art. For example, Luby I describes various forms of keying that can be employed in embodiments of the present invention.

Multi-stage coding is particularly useful where there is an expectation of data erasure or where the recipient does not begin and end reception exactly when a transmission begins and ends. The latter condition is referred to herein as "data incompleteness." Regarding erasure events, multi-stage coding shares many of the benefits of chain reaction coding described in Luby I. In particular, multi-stage output symbols are information additive, so any suitable number of packets can be used to recover an input file or stream to a desired degree of accuracy. These conditions do not adversely affect the communication process when multi-stage coding is used, because the output symbols generated with multi-stage coding are information additive. For example, if a hundred packets are lost due to a burst of noise causing data erasure, an extra hundred packets can be picked up after the burst to replace the loss of the erased packets. If thousands of packets are lost because a receiver did not tune into a transmitter when it began transmitting, the receiver could just pickup those thousands of packets from any other period of transmission, or even from another transmitter. With multi-stage coding, a receiver is not constrained to pickup any particular set of packets, so it can receive some packets from one transmitter, switch to another transmitter, lose some packets, miss the beginning or end of a given transmission and still recover an input file or block of a stream. The ability to join and leave a transmission without receiver-transmitter coordination helps to simplify the communication process.

In some embodiments, transmitting a file or stream using multi-stage coding can include generating, forming or extracting input symbols from an input file or block of a stream, computing redundant symbols, encoding input and redundant symbols into one or more output symbols, where each output symbol is generated based on its key independently of all other output symbols, and transmitting the output symbols to one or more recipients over a channel. Additionally, in some embodiments, receiving (and reconstructing) a copy of the input file or block of a stream using multi-stage coding can include receiving some set or subset of output symbols from one of more data streams, and decoding the input symbols from the values and keys of the received output symbols.

Suitable FEC erasure codes as described herein can be used to overcome the above-cited difficulties and would find use in a number of fields including multimedia broadcasting and multicasting systems and services. An FEC erasure code hereafter referred to as "a multi-stage chain reaction code" has properties that meet many of the current and future requirements of such systems and services.

Some basic properties of multi-stage chain reaction codes are that, for any packet loss conditions and for delivery of source files of any relevant size or streams of any relevant rate: (a) reception overhead of each individual receiver device ("RD") is minimized; (b) the total transmission time needed to deliver source files to any number of RDs can be minimized (c) the quality of the delivered stream to any number of RDs can be maximized for the number of output symbols sent relative to the number of input symbols, with suitable selection of transmission schedules. The RDs might be handheld devices, embedded into a vehicle, portable (i.e., movable but not typically in motion when in use) or fixed to a location.

The amount of working memory needed for decoding is low and can still provide the above properties, and the amount of computation needed to encode and decode is minimal. In this document, we provide a simple and easy to implement description of some variations of multi-stage chain reaction codes.

Multi-stage chain reaction codes are fountain codes, i.e., as many encoding packets as needed can be generated on-the-fly, each containing unique encoding symbols that are equally useful for recovering a source file or block of a stream. There are many advantages to using fountain codes versus other types of FEC codes. One advantage is that, regardless of packet loss conditions and RD availability, fountain codes minimize the number of encoding packets each RD needs to receive to reconstruct a source file or block of a stream. This is true even under harsh packet loss conditions and when, for example, mobile RDs are only intermittently turned-on or available over a long file download session.

Another advantage is the ability to generate exactly as many encoding packets as needed, making the decision on how many encoding packets to generate on-the-fly while the transmission is in progress. This can be useful if for example there is feedback from RDs indicating whether or not they received enough encoding packets to recover a source file or block of a stream. When packet loss conditions are less severe than expected the transmission can be terminated early. When packet loss conditions are more severe than expected or RDs are unavailable more often than expected the transmission can be seamlessly extended.

Another advantage is the ability to inverse multiplex. Inverse multiplexing is when a RD is able to combine received encoding packets generated at independent senders to reconstruct a source file or block of a stream. One practical use of inverse multiplexing is described in below in reference to receiving encoding packets from different senders.

Where future packet loss, RD availability and application conditions are hard to predict, it is important to choose an FEC solution that is as flexible as possible to work well under unpredictable conditions. multi-stage chain reaction codes provide a degree of flexibility unmatched by other types of FEC codes.

Aspects of the invention will now be described with reference to the figures.

System Overview

FIG. **1** is a block diagram of a communications system **100** that uses multi-stage coding. In communications system **100**, an input file **101**, or an input stream **105**, is provided to an input symbol generator **110**. Input symbol generator **110** generates a sequence of one or more input symbols (IS(**0**), IS(**1**), IS(**2**), . . . ) from the input file or stream, with each input symbol having a value and a position (denoted in FIG. **1** as a parenthesized integer). As explained above, the possible values for input symbols, i.e., its alphabet, is typically an alphabet of $2^M$ symbols, so that each input symbol codes for M bits of the input file or stream. The value of M is generally determined by the use of communication system **100**, but a general purpose system might include a symbol size input for input symbol generator **110** so that M can be varied from use to use. The output of input symbol generator **110** is provided to an encoder **115**.

Static key generator **130** produces a stream of static keys $S_0$, $S_1$, . . . . The number of the static keys generated is generally limited and depends on the specific embodiment of encoder **115**. The generation of static keys will be subsequently described in more detail. Dynamic key generator **120** generates a dynamic key for each output symbol to be generated by the encoder **115**. Each dynamic key is generated so that a large fraction of the dynamic keys for the same input file or block of a stream are unique. For example, Luby I describes embodiments of key generators that can be used. The outputs of dynamic key generator **120** and the static key generator **130** are provided to encoder **115**.

From each key I provided by dynamic key generator **120**, encoder **115** generates an output symbol, with a value B(I), from the input symbols provided by the input symbol generator. The operation of encoder **115** will be described in more detail below. The value of each output symbol is generated based on its key, on some function of one or more of the input symbols, and possibly on or more redundant symbols that had been computed from the input symbols. The collection of input symbols and redundant symbols that give rise to a specific output symbol is referred to herein as the output symbol's "associated symbols" or just its "associates". The selection of the function (the "value function") and the associates is done according to a process described in more detail below. Typically, but not always, M is the same for input symbols and output symbols, i.e., they both code for the same number of bits.

In some embodiments, the number K of input symbols is used by the encoder **115** to select the associates. If K is not known in advance, such as where the input is a streaming file, K can be just an estimate. The value K might also be used by encoder **115** to allocate storage for input symbols and any intermediate symbols generated by encoder **115**.

Encoder **115** provides output symbols to a transmit module **140**. Transmit module **140** is also provided the key of each such output symbol from the dynamic key generator **120**. Transmit module **140** transmits the output symbols, and depending on the keying method used, transmit module **140** might also transmit some data about the keys of the transmitted output symbols, over a channel **145** to a receive module **150**. Channel **145** is assumed to be an erasure channel, but that is not a requirement for proper operation of communication system **100**. Modules **140**, **145** and **150** can be any suitable hardware components, software components, physical media, or any combination thereof, so long as transmit module **140** is adapted to transmit output symbols and any needed data about their keys to channel **145** and receive module **150** is adapted to receive symbols and potentially some data about their keys from channel **145**. The value of K, if used to determine the associates, can be sent over channel **145**, or it may be set ahead of time by agreement of encoder **115** and decoder **155**.

As explained above, channel **145** can be a real-time channel, such as a path through the Internet or a broadcast link from a television transmitter to a television recipient or a telephone connection from one point to another, or channel **145** can be a storage channel, such as a CD-ROM, disk drive, Web site, or the like. Channel **145** might even be a combination of a real-time channel and a storage channel, such as a channel formed when one person transmits an input file from a personal computer to an Internet Service Provider (ISP) over a telephone line, the input file is stored on a Web server and is subsequently transmitted to a recipient over the Internet.

Because channel **145** is assumed to be an erasure channel, communications system **100** does not assume a one-to-one correspondence between the output symbols that exit receive module **150** and the output symbols that go into transmit module **140**. In fact, where channel **145** comprises a packet network, communications system **100** might not even be able to assume that the relative order of any two or more packets is preserved in transit through channel **145**. Therefore, the key of the output symbols is determined using one or more of the keying schemes described above, and not necessarily determined by the order in which the output symbols exit receive module **150**.

Receive module **150** provides the output symbols to a decoder **155**, and any data receive module **150** receives about the keys of these output symbols is provided to a dynamic key regenerator **160**. Dynamic key regenerator **160** regenerates the dynamic keys for the received output symbols and provides these dynamic keys to decoder **155**. Static key generator **163** regenerates the static keys $S_0$, $S_1$, . . . and provides them to decoder **155**. The static key generator has access to random number generator **135** used both during the encoding and the decoding process. This can be in the form of access to the same physical device if the random numbers are generated on such device, or in the form of access to the same algorithm for the generation of random numbers to achieve identical behavior. Decoder **155** uses the keys provided by dynamic key regenerator **160** and static key generator **163** together with the corresponding output symbols, to recover the input symbols (again IS(**0**), IS(**1**), IS(**2**), . . . ). Decoder **155** provides the recovered input symbols to an input file reassembler **165**, which generates a copy **170** of input file **101** or input stream **105**.

An Encoder

FIG. **2** is a block diagram of one specific embodiment of encoder **115** shown in FIG. **1**. Encoder **115** comprises a static encoder **210**, a dynamic encoder **220**, and a redundancy calculator **230**. Static encoder **210** receives the following inputs: a) original input symbols IS(**0**), IS(**1**), . . . , IS(K−1) provided by the input symbol generator **110** and stored in an input symbol buffer **205**; b) the number K of original input symbols; c) static keys $S_0$, $S_1$, . . . provided by the static key generator **130**; and d) a number R of redundant symbols. Upon receiving these inputs static encoder **205** computes R redundant symbols RE(**0**), RE(**1**), . . . , RE(R−1) as will be described below. Typically, but not always, the redundant symbols have the same size as the input symbols. In one specific embodiment, the redundant symbols generated by static encoder **210** are stored in input symbol buffer **205**. Input symbol buffer **205** may be only logical, i.e., the file or block of the stream may be physically stored in one place and the positions of the input symbols within symbol buffer **205** could only be renamings of the positions of these symbols within the original file or block of the stream.

Dynamic encoder receives the input symbols and the redundant symbols, and generates output symbols as will be described in further detail below. In one embodiment in which the redundant symbols are stored in the input symbol buffer **205**, dynamic encoder **220** receives the input symbols and redundant symbols from input symbol buffer **205**.

Redundancy calculator **230** computes the number R of redundant symbols from the number K of input symbols. This computation is described in further detail below.

Overview of Static Encoder

The general operation of static encoder **210** is shown with reference to FIGS. **3** and **4**. FIG. **3** is a simplified flow diagram illustrating one embodiment of a method of statically encoding. In a step **305**, a variable j, which keeps track of how many redundant symbols have been generated, is set to zero. Then, in a step **310**, a first redundant symbol RE(**0**) is computed as a function $F_0$ of at least some of the input symbols IS(**0**), . . . , IS(K−1). Then, in a step **315**, the variable j is incremented. Next, in a step **320**, it is tested whether all of the redundant symbols have been generated (i.e., is j greater than R−1?). If yes, then the flow ends. Otherwise, the flow proceeds to step **325**. In step **325**, RE(j) is computed as a function $F_j$ of the input symbols IS(**0**), . . . , IS(K−1) and of the previously generated redundant symbols RE(**0**), . . . , RE(j−1), where need not be a function that depends on every one of the input symbols or every one of the redundant symbols. Steps **315**, **320**, and **325** are repeated until R redundant symbols have been computed.

Referring again to FIGS. **1** and **2**, in some embodiments, static encoder **210** receives one or more static keys $S_0$, $S_1$, . . . from static key generator **130**. In these embodiments, the static encoder **210** uses the static keys to determine some or all of functions $F_0$, $F_1$, . . . $F_{j-1}$. For example, static key $S_0$ can be used to determine function $F_0$, static key $S_1$ can be used to determine function $F_1$, etc. Or, one or more of static keys $S_0$, $S_1$, . . . can be used to determine function $F_0$, one or more of static keys $S_0$, $S_1$, . . . can be used to determine function $F_1$, etc. In other embodiments, no static keys are needed, and thus static key generator **130** is not needed.

Referring now to FIGS. **2** and **3**, in some embodiments, the redundant symbols generated by static encoder **210** can be stored in input symbol buffer **205**. FIG. **4** is a simplified illustration of the operation of one embodiment of static encoder **210**. Particularly, static encoder **210** generates redundant symbol RE(j) as a function Fj of input symbols IS(**0**), . . . , IS(K−1), RE(**0**), . . . , RE(j−1), received from input symbol buffer **205**, and stores it back into input symbol buffer **205**. The exact form of the functions $F_0$, $F_1$, . . . , $F_{R-1}$ depends on the particular application. Typically, but not always, functions $F_0$, $F_1$, . . . , $F_{R-1}$ include an exclusive OR of some or all of their corresponding arguments. As described above, these functions may or may not actually employ static keys generated by static key generator **130** of FIG. **1**. For example, in one specific embodiment described below, the first few functions implement a Hamming code and do not make any use of the static keys $S_0$, $S_1$, . . . , whereas the remaining functions implement a Low-Density Parity-Check code and make explicit use of the static keys.

Overview of Multi-Stage Encoder

Referring again to FIG. **2**, dynamic encoder **220** receives input symbols IS(**0**), . . . , IS(K−1) and the redundant symbols RE(**0**), . . . , RE(R−1) and a key I for each output symbol it is to generate. The collection comprising the original input symbols and the redundant symbols will be referred to as the collection of "dynamic input symbols" hereafter. FIG. **5** is a simplified block diagram of one embodiment of a dynamic encoder, including a weight selector **510**, an associator **515**, a value function selector **520** and a calculator **525**. As shown in FIG. **5**, the K+R dynamic input symbols are stored in a dynamic symbol buffer **505**. In effect, dynamic encoder **500** performs the action illustrated in FIG. **6**, namely, to generate an output symbol value B(I) as some value function of selected input symbols.

FIG. **7** is a simplified block diagram of one specific embodiment of a static encoder according to the present invention. Static encoder **600** comprises a parameter calculator **605**, a Hamming encoder **610**, and a low-density-parity-check (LDPC) encoder **620**. Hamming encoder **610** is coupled to receive the input symbols IS(**0**), . . . , IS(K−1) from an input symbol buffer **625**, the number K of input symbols, and the parameter D. In response, Hamming encoder **610** generates D+1 redundant symbols HA(**0**), HA(**1**), . . . , HA(D) according to a Hamming code.

FIG. **8** illustrates the operation of one embodiment of the present invention that employs the static encoder shown in FIG. **7**.

FIG. **9** is a simplified flow diagram illustrating one embodiment of a parameter calculator, such as parameter calculator **605** of FIG. **7**, that calculates parameter D and E as described above. First, in a step **705**, parameter D is initialized to one. Then, in step **710**, it is determined whether $2^D$−D−1 is less than K. If no, then the flow proceeds to step **730**. If yes, the flow proceeds to step **720**, where the parameter D is incremented. Then, the flow proceeds back to step **710**. Once D has been determined, then, in step **730**, the parameter E is calculated as R−D−1.

FIG. **10** is a simplified flow diagram of such an encoder according to one embodiment of the present invention, which will now be described. First, in step **805**, a variable i is initialized to zero. Variable i keeps track of the number of redundant symbols already generated. In step **810**, a number t is calculated as the smallest odd integer greater than or equal to K/2. In step **815**, values $P_1$, $P_2$, . . . , $P_t$ are generated based on K, t, and a static key $S_i$. The values $P_1$, $P_2$, . . . , $P_t$ indicate the positions of input symbols that will be used to generate a redundant symbol. In one particular embodiment, an associator such as associator **515** of FIG. **5** is used to generate $P_1$, $P_2$, . . . , $P_t$. In particular, the value t can be provided as the W(I) input, the value K can be provided as the K+R input, and the static key $S_i$ can be provided as the key I input. It should be noted that many different values oft would yield similar coding effects, and thus this particular choice is only an example. In step **820**, the value of RE(i) is computed as the XOR of the values IS($P_1$), IS($P_2$), . . . , IS($P_t$). In step **825**, the variable i is incremented by one to prepare computation of the next redundant symbol, and in step **830**, it is determined whether all the redundant symbols have been computed. If not, then the flow returns to step **815**.

FIG. **11** is a simplified block diagram illustrating one embodiment of a decoder according to the present invention. Decoder **900** can be used, for example, to implement decoder **155** of FIG. **1**.

Decoder **900** comprises a dynamic decoder **905** and a static decoder **910**. Input symbols and redundant symbols recovered by dynamic decoder **905** are stored in a reconstruction buffer **915**. Upon completion of dynamic decoding, static decoder **910** attempts to recover any input symbols not recovered by dynamic decoder **905**, if any. In particular, static decoder **910** receives input symbols and redundant symbols from reconstruction buffer **915**.

FIG. **12** is a simplified flow diagram illustrating one embodiment of a method for decoding according to the present invention. In step **1005**, Q output symbols are received by the decoder. The value of Q can depend on the number of input symbols and the specific dynamic encoder used. The value of Q can also depend on the desired degree of accuracy to which the decoder can recover the input symbols. For example, if it is desired that the decoder can recover all of the input symbols with a high probability, then Q should be chosen to be larger than the number of input symbols. Particularly, in some applications, when the number of input symbols is large, Q can be less than 3% larger than the number of original input symbols. In other applications, when the number of input symbols is small, Q can be at least 10% larger than the number of input symbols. Specifically, Q can be chosen as the number K of input symbols plus a number A, where A is chosen to ensure that the decoder can regenerate all of the input symbols with a high probability. Determination of the number A is described in more detail below. If it is acceptable for the decoder to be unable to decode all of the input symbols (either sometimes or always), then Q can be less than K+A, equal to K, or even less than K. Clearly, one aim of an overall coding system will often be to decrease the number Q as much as possible, while maintaining good probabilistic guarantees on the success of the decoding process with respect to the desired degree of accuracy.

In step **1010**, dynamic decoder **905** regenerates input symbols and redundant symbols from the Q received output symbols. It is to be understood, that steps **1005** and **1010** can be performed substantially concurrently. For example, dynamic decoder **905** can begin regenerating input symbols and redundant symbols prior to the decoder receiving Q output symbols.

After dynamic decoder **905** has processed Q output symbols, then it is determined whether the input symbols have been recovered to a desired degree of accuracy. The desired degree of accuracy may be, for example, all of the input symbols, or some number, percentage, etc., less than all of the input symbols. If yes, then the flow ends. If no, then the flow proceeds to step **1020**. In step **1020**, static decoder **910** attempts to recover any input symbols that dynamic decoder **905** was unable to recover. After static encoder **910** has processed the input symbols and redundant symbols recovered by dynamic encoder **905**, then the flow ends.

FIG. **13** is a simplified flow diagram illustrating another embodiment of a method for decoding according to the present invention. This embodiment is similar to that described with respect to FIG. **11**, and includes steps **1005**, **1010**, **1015**, and **1025** in common. But, after step **1025**, the flow proceeds to step **1030**, in which it is determined whether the input symbols have been recovered to a desired degree of accuracy. If yes, then the flow ends. If no, then the flow proceeds to step **1035**. In step **1035**, one or more additional output symbols are received. Then, the flow proceeds back to step **1010**, so that dynamic decoder **905** and/or static decoder **910** can attempt to recover the remaining unrecovered input symbols.

FIG. **14** is a simplified flow diagram illustrating yet another embodiment of a method for decoding according to the present invention. In step **1055**, output symbols are received by the decoder, and in step **1060**, dynamic decoder **905** regenerates input symbols and redundant symbols from the received output symbols. Then, in step **1065**, it is determined whether dynamic decoding should be ended. This determination can be based on one or more of the number of output symbols processed, the number of input symbols recovered, the current rate at which additional input symbols are being recovered, the time spent processing output symbols, etc.

In step **1065**, if it is determined that dynamic decoding is not to be stopped, then the flow proceeds back to step **1055**. But, if in step **1065**, it is determined to end dynamic decoding, then the flow proceeds to step **1070**. In step **1070**, it is determined whether the input symbols have been recovered to a desired degree of accuracy. If yes, then the flow ends. If no, then the flow proceeds to step **1075**. In step **1075**, static decoder **910** attempts to recover any input symbols that dynamic decoder **905** was unable to recover. After static encoder **910** has processed the input symbols and redundant symbols recovered by dynamic encoder **905**, the flow ends.

FIG. **15** shows one embodiment of dynamic decoder according to the present invention. Dynamic decoder **1100** includes similar components as those of dynamic encoder **500** shown in FIG. **5**. Decoder **1100** is similar to embodiments of chain reaction decoders described in Luby I and Luby II. Dynamic decoder **1100** comprises a weight selector **510**, an associator **515**, a value function selector **520**, an output symbol buffer **1105**, a reducer **1115**, a reconstructor **1120** and a reconstruction buffer **1125**.

FIG. **16** is a simplified block diagram illustrating one embodiment of a static decoder. This embodiment can be used when the data is encoded with a static encoder such as described with reference to FIG. **7**. Static decoder **1200** comprises a LDPC decoder **1205** and a Hamming decoder **1210**. The LDPC decoder **1205** receives input symbols and redundant symbols from a reconstruction buffer **1215**, and attempts to reconstruct those symbols of reconstruction buffer **1215** unrecovered after the decoding step of the dynamic decoder. In some embodiments, reconstruction buffer **1215** is reconstruction buffer **1125** (FIG. **15**).

Many variations of LDPC decoders and Hamming decoders are well known to those skilled in the art, and can be employed in various embodiments according to the present invention. In one specific embodiment, Hamming decoder is implemented using a Gaussian elimination algorithm. Many variations of Gaussian elimination algorithms are well known to those skilled in the art, and can be employed in various embodiments according to the present invention.

Variations

Multi-stage chain reaction codes as described above are not systematic codes, i.e., all of the original source symbols of a source block are not necessarily among the encoding symbols that are sent. However, systematic FEC codes are useful for a file download system or service, and very important for a streaming system or service. As shown in the implementation below, a modified code can be made to be systematic and still maintain the fountain code and other described properties.

One reason why it is easy to architect a variety of supplemental services using multi-stage codes is that it can combine received encoding symbols from multiple senders to reconstruct a source file or stream without coordination among the senders. The only requirement is that the senders use differing sets of keys to generate the encoding symbols that they send in encoding packets to the code. Ways to achieve this include designating different ranges of the key space to be used by each such sender, or generating keys randomly at each sender.

As an example of the use of this capability, consider providing a supplemental service to a file download service that allows multi-stage chain reaction codes that did not receive enough encoding packets to reconstruct a source file from the file download session to request additional encoding packets to be sent from a make-up sender, e.g., via a HTTP session. The make-up sender generates encoding symbols from the source file and sends them, for example using HTTP, and all these encoding symbols can be combined with those received from the file download session to recover the source file. Using this approach allows different senders to provide incremental source file delivery services without coordination between the senders, and ensuring that each individual receiver need receive only a minimal number of encoding packets to recover each source file.

Implementations of Various Stages of Multi-Stage Codes

FEC Scheme Definition

A packet using these techniques might be represented with header information such as an FEC Payload ID of four octets comprising a Source Block Number (SBN) (16 bit integer identifier for the source block that the encoding symbols within the packet relate to) and an Encoding Symbol ID (ESI) (16 bit integer identifier for the encoding symbols within the packet). One suitable interpretation of the Source Block Number and Encoding Symbol Identifier is defined in Sections B below. FEC Object Transmission information might comprise the FEC Encoding ID, a Transfer Length (F) and the parameters T, Z, N and A defined in below. The parameters T and Z are 16 bit unsigned integers, N and A are 8 bit unsigned integers.

An FEC encoding scheme for MBMS forward error correction is defined in the sections below. It defines two different FEC Payload ID formats, one for FEC source packets and another for FEC repair packets, but variations for nonsystematic codes are also possible.

The Source FEC payload ID might comprise a Source Block Number (SBN) (16 bit integer identifier for the source block that the encoding symbols within the packet relate to) and an Encoding Symbol ID (ESI) (16 bit integer identifier for the encoding symbols within the packet), while the Repair FEC Payload ID might comprise a Source Block Number (SBN) (16 bit integer identifier for the source block that the repair symbols within the packet relate to), an Encoding Symbol ID (ESI) (16 bit integer identifier for the repair symbols within the packet), and a Source Block Length (SBL) (16 bits, representing the number of source symbols in the source block. The interpretation of the Source Block Number, Encoding Symbol Identifier and Source Block Length is defined below.

FEC Object Transmission information might comprise the FEC Encoding ID, the maximum source block length, in symbols, and the symbol size, in bytes. The symbol size and maximum source block length might comprise a four octet field of Symbol Size (T) (16 bits representing the size of an encoding symbol, in bytes), and a Maximum Source Block Length (16 bits representing the maximum length of a source block, in symbols).

The sections below specify the systematic MSCR forward error correction code and its application to MBMS and other uses. MSCR is a fountain code, i.e., as many encoding symbols as needed can be generated by the encoder on-the-fly from the source symbols of a block. The decoder is able to recover the source block from any set of encoding symbols only slightly more in number than the number of source symbols. The code described in this document is a systematic code, that is, the original source symbols are sent unmodified from sender to receiver, as well as a number of repair symbols.

B.1 Definitions, Symbols and Abbreviations

B.1.1 Definitions

For the purposes of this description, the following terms and definitions apply.

Source block: a block of K source symbols which are considered together for MSCR encoding purposes.

Source symbol: the smallest unit of data used during the encoding process. All source symbols within a source block have the same size.

Encoding symbol: a symbol that is included in a data packet. The encoding symbols comprise the source symbols and the repair symbols. Repair symbols generated from a source block have the same size as the source symbols of that source block.

Systematic code: a code in which the source symbols are included as part of the encoding symbols sent for a source block.

Repair symbol: the encoding symbols sent for a source block that are not the source symbols. The repair symbols are generated based on the source symbols.

Intermediate symbols: symbols generated from the source symbols using an inverse encoding process. The repair symbols are then generated directly from the intermediate symbols. The encoding symbols do not include the intermediate symbols, i.e., intermediate symbols are not included in data packets.

Symbol: a unit of data. The size, in bytes, of a symbol is known as the symbol size.

Encoding symbol group: a group of encoding symbols that are sent together, i.e., within the same packet whose relationship to the source symbols can be derived from a single Encoding Symbol ID.

Encoding Symbol ID: information that defines the relationship between the symbols of an encoding symbol group and the source symbols.

Encoding packet: data packets that contain encoding symbols

Sub-block: a source block is sometime broken into sub-blocks, each of which is sufficiently small to be decoded in working memory. For a source block comprising K source symbols, each sub-block comprises K sub-symbols, each symbol of the source block being composed of one sub-symbol from each sub-block.

Sub-symbol: part of a symbol. Each source symbol is composed of as many sub-symbols as there are sub-blocks in the source block.

Source packet: data packets that contain source symbols.

Repair packet: data packets that contain repair symbols.

B.1.2. Symbols

| | |
|---|---|
| i, j, x, h, a, b, d, v, m | represent positive integers |
| ceil(x) | denotes the smallest positive integer which is greater than or equal to x |
| choose(i, j) | denotes the number of ways j objects can be chosen from among i objects without repetition |
| floor(x) | denotes the largest positive integer which is less than or equal to x |
| i % j | denotes i modulo j |
| X ^ Y | denotes, for equal-length bit strings X and Y, the bitwise exclusive-or of X and Y |
| A | denote a symbol alignment parameter. Symbol and sub-symbol sizes are restricted to be multiples of A. |
| $A^T$ | denotes the transposed matrix of matrix A |
| $A^{-1}$ | denotes the inverse matrix of matrix A |
| K | denotes the number of symbols in a single source block |
| $K_{MAX}$ | denotes the maximum number of source symbols that can be in a single source block. Set to 8192. |
| L | denotes the number of pre-coding symbols for a single source block |
| S | denotes the number of LDPC symbols for a single source block |
| H | denotes the number of Half symbols for a single source block |
| C | denotes an array of intermediate symbols, C[0], C[1], C[2], . . . , C[L − 1] |
| C' | denotes an array of source symbols, C'[0], C'[1], C'[2], . . . , C'[K − 1] |
| X | a non-negative integer value |
| $V_0$, $V_1$ | two arrays of 4-byte integers, $V_0[0], V_0[1], \ldots, V_0[255]$ and $V_1[0], V_1[1], \ldots, V_1[255]$ |
| Rand[X, i, m] | a pseudo-random number generator |

-continued

| | |
|---|---|
| Deg[v] | a degree generator |
| LTEnc[K, C, (d, a, b)] | a LT encoding symbol generator |
| Trip[K, X] | a triple generator function |
| G | the number of symbols within an encoding symbol group |
| N | the number of sub-blocks within a source block |
| T | the symbol size in bytes. If the source block is partitioned into sub-blocks, then T = T' · N. |
| T' | the sub-symbol size, in bytes. If the source block is not partitioned into sub-blocks then T' is not relevant. |
| F | the file size, for file download, in bytes |
| I | the sub-block size in bytes |
| P | for file download, the payload size of each packet, in bytes, that is used in the recommended derivation of the file download transport parameters. For streaming, the payload size of each repair packet, in bytes, that is used in the recommended derivation of the streaming transport parameters. |
| Q | Q = 65521, i.e., Q is the largest prime smaller than $2^{16}$ |
| Z | the number of source blocks, for file download |
| J(K) | the systematic index associated with K |
| G | denotes any generator matrix |
| $I_S$ | denotes the S×S identity matrix |
| $0_{S\times H}$ | denotes the S×H zero matrix |

B.1.3 Abbreviations

For the purposes of the present document, the following abbreviations apply:

| | |
|---|---|
| ESI | Encoding Symbol ID |
| LDPC | Low Density Parity Check |
| LT | Luby Transform |
| SBN | Source Block Number |
| SBL | Source Block Length (in units of symbols) |

B.2. Overview

The MSCR forward error correction code can be applied to both MBMS file delivery and MBMS streaming applications. MSCR code aspects which are specific to each of these applications are discussed in Sections B.3 and B.4 of this document.

A component of the systematic MSCR code is the basic encoder described in Section B.5. First, it is described how to derive values for a set of intermediate symbols from the original source symbols such that knowledge of the intermediate symbols is sufficient to reconstruct the source symbols. Secondly, the encoder produces repair symbols which are each the exclusive OR of a number of the intermediate symbols. The encoding symbols are the combination of the source and repair symbols. The repair symbols are produced in such a way that the intermediate symbols and therefore also the source symbols can be recovered from any sufficiently large set of encoding symbols.

This document defines the systematic MSCR code encoder. A number of possible decoding algorithms are possible. An efficient decoding algorithm is provided in Section B.6.

The construction of the intermediate and repair symbols is based in part on a pseudo-random number generator described in Section B.5. This generator is based on a fixed set of 512 random numbers that are available to both sender and receiver. An example set of numbers are those provided in Appendix B.1.

Finally, the construction of the intermediate symbols from the source symbols is governed by a "'systematic index". An example set of values for the systematic index is shown in Appendix A for source block sizes from 4 source symbols to $K_{MAX}$=8192 source symbols.

B.3. File Download

B.3.1. Source Block Construction

B.3.1.1. General

In order to apply the MSCR encoder to a source file, the file may be broken into Z≥1 blocks, known as source blocks. The MSCR encoder is applied independently to each source block. Each source block is identified by a unique integer Source Block Number (SBN), where the first source block has SBN zero, the second has SBN one, etc. Each source block is divided into a number, K, of source symbols of size T bytes each. Each source symbol is identified by a unique integer Encoding Symbol Identifier (ESI), where the first source symbol of a source block has ESI zero, the second has ESI one, etc.

Each source block with K source symbols is divided into N≥1 sub-blocks, which are small enough to be decoded in the working memory. Each sub-block is divided into K sub-symbols of size T'.

Note that the value of K is not necessarily the same for each source block of a file and the value of T' may not necessarily be the same for each sub-block of a source block. However, the symbol size T is the same for all source blocks of a file and the number of symbols, K is the same for every sub-block of a source block. Exact partitioning of the file into source blocks and sub-blocks is described in B.3.1.2 below.

FIG. **17** shows an example source block placed into a two dimensional array, where each entry is a T'-byte sub-symbol, each row is a sub-block and each column is a source symbol. In this example, the value of T' is the same for every sub-block. The number shown in each sub-symbol entry indicates their original order within the source block. For example, the sub-symbol numbered K contains bytes T'. K through T'·(K+1)−1 of the source block. Then, source symbol i is the concatenation of the ith sub-symbol from each of the sub-blocks, which corresponds to the sub-symbols of the source block numbered i, K+i, 2·K+i, . . . , (N−1)·K+i.

B.3.1.2 Source Block and Sub-Block Partitioning

The construction of source blocks and sub-blocks is determined based on five input parameters, F, A, T, Z and N and a function Partition[ ]. The five input parameters are defined as follows:

F the size of the file, in bytes

A a symbol alignment parameter, in bytes

T the symbol size, in bytes, which must be a multiple of A

Z the number of source blocks

N the number of sub-blocks in each source block

These parameters might be set so that ceil(ceil(F/T)/Z) $\leq K_{MAX}$. Recommendations for derivation of these parameters are provided in Section B.3.4.

The function Partition[ ] takes a pair of integers (I, J) as input and derives four integers ($I_L$, $I_S$, $J_L$, $J_S$) as output. Specifically, the value of Partition[I, J] is a sequence of four integers ($I_L$, $I_S$, $J_L$, $J_S$), where $I_L$=ceil(I/J), $I_S$=floor(I/J), $J_L=I-I_S\cdot J$ and $J_S=J-J_L$. Partition[ ] derives parameters for partitioning a block of size I into J approximately equal sized blocks. Specifically, $J_L$ blocks of length $I_L$ and $J_S$ blocks of length $I_S$.

The source file might be partitioned into source blocks and sub-blocks as follows:

Let,

$K_t$=ceil(F/T)

($K_L$, $K_S$, $Z_L$, $Z_S$)=Partition[$K_t$, Z]

($T_L$, $T_S$, $N_L$, $N_S$)=Partition[T/A, N]

Then, the file might be partitioned into $Z=Z_L+Z_S$ contiguous source blocks, the first $Z_L$ source blocks each having length $K_L\cdot T$ bytes and the remaining $Z_S$ source blocks each having $K_S\cdot T$ bytes.

If $K_t\cdot T>F$ then for encoding purposes, the last symbol might be padded at the end with $K_t\cdot T-F$ zero bytes.

Next, each source block might be divided into $N=N_L+N_S$ contiguous sub-blocks, the first $N_L$ sub-blocks each comprising K contiguous sub-symbols of size of $T_L\cdot A$ and the remaining $N_S$ sub-blocks each comprising K contiguous sub-symbols of size of $T_S\cdot A$. The symbol alignment parameter A ensures that sub-symbols are always a multiple of A bytes.

Finally, the mth symbol of a source block comprises the concatenation of the mth sub-symbol from each of the N sub-blocks.

B.3.2. Encoding Packet Construction

B.3.2.1. General

Each encoding packet contains the following information:

Source Block Number (SBN)

Encoding Symbol ID (ESI)

encoding symbol(s)

Each source block is encoded independently of the others. Source blocks are numbered consecutively from zero.

Encoding Symbol ID values from 0 to K−1 identify the source symbols. Encoding Symbol IDs from K onwards identify repair symbols.

B.3.2.2 Encoding Packet Construction

Each encoding packet preferably either consists entirely of source symbols (source packet) or entirely of repair symbols (repair packet). A packet may contain any number of symbols from the same source block. In the case that the last symbol in the packet includes padding bytes added for FEC encoding purposes then these bytes need not be included in the packet. Otherwise, only whole symbols might be included.

The Encoding Symbol ID, X, carried in each source packet is the Encoding Symbol ID of the first source symbol carried in that packet. The subsequent source symbols in the packet have Encoding Symbol IDs, X+1 to X+G−1, in sequential order, where G is the number of symbols in the packet.

Similarly, the Encoding Symbol ID, X, placed into a repair packet is the Encoding Symbol ID of the first repair symbol in the repair packet and the subsequent repair symbols in the packet have Encoding Symbol IDs X+1 to X+G−1 in sequential order, where G is the number of symbols in the packet.

Note that it is not necessary for the receiver to know the total number of repair packets. The G repair symbol triples (d[0], a[0], b[0]), . . . , (d[G−1], a[G−1], b[G−1]) for the repair symbols placed into a repair packet with ESI X are computed using the Triple generator defined in B.5.3.4 as follows:

For each i=0, . . . , G−1

(d[i], a[i], b[i])=Trip[K,X+i]

The G repair symbols to be placed in repair packet with ESI X are calculated based on the repair symbol triples as described in Section B.5.3 using the intermediate symbols C and the LT encoder LTenc[K, C, (d[i], a[i], b[i])].

B.3.3. Transport

This section describes the information exchange between the MSCR encoder/decoder and any transport protocol making use of MSCR forward error correction for file delivery.

The MSCR encoder and decoder for file delivery require the following information from the transport protocol: the file size, F, in bytes, the symbol alignment parameter, A, the symbol size, T, in bytes, which is a multiple of A, the number of source blocks, Z, the number of sub-blocks in each source block, N. The MSCR encoder for file delivery additionally requires the file to be encoded, F bytes.

The MSCR encoder supplies the transport protocol with encoding packet information comprising, for each packet, the SBN, the ESI and the encoding symbol(s). The transport protocol might communicate this information transparently to the MSCR decoder.

B.3.4. Recommended Parameters (Informative)

B.3.4.1 Parameter Derivation Algorithm

This section provides recommendations for the derivation of the four transport parameters, A, T, Z and N. This recommendation is based on the following input parameters:

F the file size, in bytes

W a target on the sub-block size, in bytes

P the maximum packet payload size, in bytes, which is assumed to be a multiple of A

A the symbol alignment factor, in bytes

$K_{MAX}$ the maximum number of source symbols per source block.

$K_{MIN}$ a minimum target on the number of symbols per source block

$G_{MAX}$ a maximum target number of symbols per packet

Based on the above inputs, the transport parameters T, Z and N are calculated as follows:

Let,

G=min{ceil($P\cdot K_{MIN}$/F), P/A, $G_{MAX}$}—the approximate number of symbols per packet

T=floor(P/(A·G))·A

$K_t$=ceil(F/T)—the total number of symbols in the file

Z=ceil($K_t/K_{MAX}$)

N=min{ceil(ceil($K_t$/Z)·T/W), T/A}

The values of G and N derived above should be considered as lower bounds. It may be advantageous to increase these values, for example to the nearest power of two. In particular, the above algorithm does not guarantee that the symbol size, T, divides the maximum packet size, P, and so it may not be possible to use the packets of size exactly P. If, instead, G is chosen to be a value which divides P/A, then the symbol size, T, will be a divisor of P and packets of size P can be used.

Recommended settings for the input parameters, W, A, $K_{MIN}$ and $G_{MAX}$ are as follows:

W=256 KB A=4 $K_{MIN}$=1024 $G_{MAX}$=10

B.3.4.2 Examples

The above algorithm leads to transport parameters as shown in FIG. **18**, assuming the recommended values for W, A, $K_{MIN}$ and $G_{MAX}$ and P=512.

B.4. Streaming

B.4.1. Source Block Construction

A source block is constructed by the transport protocol, for example as defined in this document, making use of the Systematic MSCR Forward Error Correction code. The symbol size, T, to be used for source block construction and the repair symbol construction are provided by the transport protocol. The parameter T might be set so that the number of source symbols in any source block is at most $K_{MAX}$.

Recommended parameters are presented in section B.4.4.

B.4.2. Encoding Packet Construction

As described in B.4.3., each repair packet contains the SBN, ESI, SBL and repair symbol(s). The number of repair symbols contained within a repair packet is computed from the packet length. The ESI values placed into the repair packets and the repair symbol triples used to generate the repair symbols are computed as described in Section B.3.2.2.

B.4.3. Transport

This section describes the information exchange between the MSCR encoder/decoder and any transport protocol making use of MSCR forward error correction for streaming. The MSCR encoder for streaming might use the following information from the transport protocol for each source block: the symbol size, T, in bytes, the number of symbols in the source block, K, the Source Block Number (SBN) and the source symbols to be encoded, KT bytes. The MSCR encoder supplies the transport protocol with encoding packet information comprising, for each repair packet, the SBN, the ESI, the SBL and the repair symbol(s). The transport protocol might communicate this information transparently to the MSCR decoder.

B.4.4. Recommended Parameters

B.4.4.1 Parameter Derivation Algorithm

This section provides recommendations for the derivation of the transport parameter T. This recommendation is based on the following input parameters:

| | |
|---|---|
| B | the maximum source block size, in bytes |
| P | the maximum repair packet payload size, in bytes, which is a multiple of A |
| A | the symbol alignment factor, in bytes |
| $K_{MAX}$ | the maximum number of source symbols per source block. |
| $K_{MIN}$ | a minimum target on the number of symbols per source block |
| $G_{MAX}$ | a maximum target number of symbols per repair packet |

A requirement on these inputs is that ceil(B/P)≤$K_{MAX}$. Based on the above inputs, the transport parameter T is calculated as follows:

Let G=min {ceil(P·$K_{MIN}$/B), P/A, $G_{MAX}$}—the approximate number of symbols per packet

$$T=\text{floor}(P/(A\cdot G))\cdot A$$

The value of T derived above should be considered as a guide to the actual value of T used. It may be advantageous to ensure that T divides into P, or it may be advantageous to set the value of T smaller to minimize wastage when full size repair symbols are used to recover partial source symbols at the end of lost source packets (as long as the maximum number of source symbols in a source block does not exceed $K_{MAX}$). Furthermore, the choice of T may depend on the source packet size distribution, e.g., if all source packets are the same size then it is advantageous to choose T so that the actual payload size of a repair packet P', where P' is a multiple of T, is equal to (or as few bytes as possible larger than) the number of bytes each source packet occupies in the source block.

Recommended settings for the input parameters, A, $K_{MIN}$ and $G_{MAX}$ are as follows:

A=4 $K_{MIN}$=1024 $G_{MAX}$=10

B.4.4.2 Examples

The above algorithm leads to transport parameters as shown in FIG. **19**, assuming the recommended values for A, $K_{MIN}$, and $G_{MAX}$ and P=512.

B.5. Systematic MSCR Encoder

B.5.1. Encoding Overview

The systematic MSCR encoder is used to generate repair symbols from a source block that comprises K source symbols.

Symbols are the fundamental data units of the encoding and decoding process. For each source block (sub-block) all symbols (sub-symbols) are the same size. The atomic operation performed on symbols (sub-symbols) for both encoding and decoding is the exclusive-or operation.

Let C'[0], . . . , C'[K−1] denote the K source symbols.

Let C[0], . . . , C[L−1] denote L intermediate symbols.

The first step of encoding is to generate a number, L>K, of intermediate symbols from the K source symbols. In this step, K source triples (d[0], a[0], b[0]), . . . , (d[K−1], a[K−1], b[K−1]) are generated using the Trip[ ] generator as described in Section B.5.4.4. The K source triples are associated with the K source symbols and are then used to determine the L intermediate symbols C[0], . . . , C[L−1] from the source symbols using an inverse encoding process. This process can be can be realized by a MSCR decoding process.

Certain "pre-coding relationships" must hold within the L intermediate symbols. Section B.5.2 describes these relationships and how the intermediate symbols are generated from the source symbols.

Once the intermediate symbols have been generated, repair symbols are produced and one or more repair symbols are placed as a group into a single data packet. Each repair symbol group is associated with an Encoding Symbol ID (ESI) and a number, G, of encoding symbols. The ESI is used to generate a triple of three integers, (d, a, b) for each repair symbol, again using the Trip[ ] generator as described in Section B.5.4.4. This is done as described in Sections B.3 and B.4 using the generators described in Section B.5.4. Then, each (d,a,b)-triple is used to generate the corresponding repair symbol from the intermediate symbols using the LTEnc[K, C[0], . . . , C[L−1], (d,a,b)] generator described in Section B.5.4.3.

B.5.2. First Encoding Step: Intermediate Symbol Generation

B.5.2.1General

The first encoding step is a pre-coding step to generate the L intermediate symbols C[0], . . . , C[L−1] from the source symbols C'[0], . . . , C'[K−1]. The intermediate symbols are uniquely defined by two sets of constraints:

1. The intermediate symbols are related to the source symbols by a set of source symbol triples. The generation of the source symbol triples is defined in Section B.5.2.2 using the Trip[ ] generator as described in Section B.5.4.4.

2. A set of pre-coding relationships hold within the intermediate symbols themselves. These are defined in Section B.5.2.3.

The generation of the L intermediate symbols is then defined in Section 5.2.4.

B.5.2.2 Source Symbol Triples

Each of the K source symbols is associated with a triple (d[i], a[i], b[i]) for 0≤i<K. The source symbol triples are determined using the Triple generator defined in Section B.5.4.4 as:

For each i, 0≤i<K

(d[i], a[i], b[i])=Trip[K, i]

8.5.2.3 Pre-Coding Relationships

The pre-coding relationships amongst the L intermediate symbols are defined by expressing the last L-K intermediate symbols in terms of the first K intermediate symbols.

The last L-K intermediate symbols C[K], . . . , C[L−1] comprise S LDPC symbols and H Half symbols The values of S and H are determined from K as described below. Then L=K+S+H.

Let

X be the smallest positive integer such that $X\cdot(X-1)=2\cdot K$.

S be the smallest prime integer such that $S\geq\text{ceil}(0.01\cdot K)+X$

H be the smallest integer such that choose (H, ceil(H/2)) ≥K+S

H'=ceil(H/2)L=K+S+H

C[0], . . . , C[K−1] denote the first K intermediate symbols

C[K], . . . , C[K+S−1] denote the S LDPC symbols, initialised to zero

C[K+S], . . . , C[L−1] denote the H Half symbols, initialised to zero

The S LDPC symbols are defined to be the values of C[K], . . . , C[K+S−1] at the end of the following process:

For i=0, . . . , K−1 do

a=1+(floor(i/S) % (S−1))

b=i % S

C[K+b]=C[K+b]^C[i]

b=(b+a) % S

C[K+b]=C[K+b]^C[i]

b=(b+a) % S

C[K+b]=C[K+b]^C[i]

The H Half symbols are defined as follows:

Let

g[i]=i ^(floor(i/2)) for all positive integers i

Note: g[i] is the Gray sequence, in which each element differs from the previous one in a single bit position

g[j, k] denote the $j^{th}$ element, j=0, 1, 2, . . . , of the subsequence of g[i] whose elements have exactly k non-zero bits in their binary representation

Then, the Half symbols are defined as the values of C[K+S], . . . , C[L−1] after the following process:

For h=0, . . . , H−1 do

For j=0, . . . , K+S−1 do

If bit h of g[j,H'] is equal to 1 then C[h+K+S]=C[h+K+S]^ C[j].

B.5.2.4 Intermediate Symbols

B.5.2.4.1 Definition

Given the K source symbols C'[0], C'[1], . . . , C'[K−1] the L intermediate symbols C[0], C[1], . . . , C[L−1] are the uniquely defined symbol values that satisfy the following conditions:

1. The K source symbols C'[0], C'[1], . . . , C'[K−1] satisfy the K constraints

C'[i]≡LTEnc[K, (C[0], . . . , C[L−1]), (d[i], a[i], b[i])], for all i, 0≤i<K.

2. The L intermediate symbols C[0], C[1], . . . , C[L−1] satisfy the pre-coding relationships defined in B.5.2.3.

B.5.2.4.2 Calculation of Intermediate Symbols

This subsection describes a possible method for calculation of the L intermediate symbols C[0], C[1], . . . , C[L−1] satisfying the constraints in B.5.2.4.1

The generator matrix G for a code which generates N output symbols from K input symbols is an N×K matrix over GF(2), where each row corresponds to one of the output symbols and each column to one of the input symbols and where the $i^{th}$ output symbol is equal to the sum of those input symbols whose column contains a non-zero entry in row i.

Then, the L intermediate symbols can be calculated as follows:

Let

C denote the column vector of the L intermediate symbols, C[0], C[1], . . . , C[L−1].



D denote the column vector comprising S+H zero symbols followed by the K source symbols C'[0], C'[1], C'[K−1]

Then the above constraints define an L×L matrix over GF(2), A, such that:

A·C=D

The matrix A can be constructed as follows:

Let:

$G_{LDPC}$ be the S×K generator matrix of the LDPC symbols. So,

$G_{LDPC}\cdot(C[0], \ldots, C[K-1])^T=(C[K], \ldots, C[K+S-1])^T$

$G_{Half}$ be the H×(K+S) generator matrix of the Half symbols, So,

$G_{Half}\cdot(C[0], \ldots, C[S+K-1])^T=(C[K+S], \ldots, C[K+S+H-1])^T$

$I_S$ be the S×S identity matrix

$I_H$ be the H×H identity matrix

$0_{S\times H}$ be the S×H zero matrix

$G_{LT}$ be the K×L generator matrix of the encoding symbols generated by the LT Encoder.

So,

$G_{LT}\cdot(C[0], \ldots, C[L-1])^T=(C'[1], \ldots, C'[K-1])^T$

i.e. $G_{LTi,j}=1$ if and only if C[i] is included in the symbols which are XORed to produce LTEnc[K, (C[0], . . . , C[L−1]), (d[i], a[i], b[i])].

Then:

The first S rows of A are equal to $G_{LDPC}|I_S|Z_{S\times H}$.

The next H rows of A are equal to $G_{Half}|I_H$.

The remaining K rows of A are equal to $G_{LT}$.

The matrix A is depicted in FIG. **20**. The intermediate symbols can then be calculated as:

$$C=A^{-1}\cdot D$$

The source triples are generated such that for any K matrix A has full rank and is therefore invertible. This calculation can be realized by applying a MSCR decoding process to the K source symbols C'[0], C'[1], . . . , C'[K−1] to produce the L intermediate symbols C[0], C[1], . . . , C[L−1].

To efficiently generate the intermediate symbols from the source symbols, it is recommended that an efficient decoder implementation such as that described in Section B.6 be used. The source symbol triples are designed to facilitate efficient decoding of the source symbols using that algorithm.

B.5.3. Second Encoding Step: LT Encoding

In the second encoding step, the repair symbol with ESI X is generated by applying the generator LTEnc[K, (C[0], C[1], . . . , C[L−1]), (d, a, b)] defined in Section B.5.4 to the L intermediate symbols C[0], C[1], . . . , C[L−1] using the triple (d, a, b)=Trip[K,X] generated according to Sections B.3.2.2 and B.4.2.

B.5.4. Generators

B.5.4.1 Random Generator

The random number generator Rand[X, i, m] is defined as follows, where X is a non-negative integer, i is a non-negative integer and m is a positive integer and the value produced is an integer between 0 and m−1. Let $V_0$ and $V_1$ be arrays of 256 entries each, where each entry is a 4-byte unsigned integer. These arrays are provided in Section B.7.

Then,

Rand[X, i, m]=($V_0$[(X+i) % 256]^$V_1$[(floor(X/256)+i) % 256]) % m

B.5.4.2 Degree Generator

The degree generator Deg[v] is defined as follows, where v is an integer that is at least 0 and less than $2^{20}$=1048576.

In FIG. **21**, find the index j such that f[j−1]≤v<f[j]

Deg[v]=d[j]

B.5.4.3 LT Encoding Symbol Generator

The encoding symbol generator LTEnc[K, (C[0], C[1], . . . , C[L−1]), (d, a, b)] takes the following inputs:

K is the number of source symbols (or sub-symbols) for the source block (sub-block). Let L be derived from K as described in Section B.5.2, and let L' be the smallest prime integer greater than or equal to L.

(C[0], C[1], . . . , C[L−1]) is the array of L intermediate symbols (sub-symbols) generated as described in Section B.5.2

(d, a, b) is a source triple determined using the Triple generator defined in Section B.5.3.4, whereby d is an integer denoting an encoding symbol degree, a is an integer between 1 and L'−1 inclusive and b is an integer between 0 and L'−1 inclusive.

The encoding symbol generator produces a single encoding symbol as output, according to the following algorithm:

While (b≥L) do b=(b+a) % L'

LTEnc[K, (C[0], C[1], . . . , C[L−1]), (d, a, b)]=C[b].

For j=1, . . . , min(d−1, L−1) do

b=(b+a) % L'

While (b≥L) do b=(b+a) % L'

LTEnc[K, (C[0], C[1], . . . , C[L−1]), (d, a, b)]=LTEnc[K, (C[0], C[1], . . . , C[L−1]), (d, a, b)]^C[b]

B.5.4.4 Triple Generator

The triple generator Trip[K,X] takes the following inputs:

K The number of source symbols

X An encoding symbol ID

Let

L be determined from K as described in Section B.5.2

L' be the smallest prime that is greater than or equal to L

Q=65521, the largest prime smaller than $2^{16}$.

J(K) be the systematic index associated with K, as defined in Appendix A

The output of the triple generator is a triples, (d, a, b) determined as follows:

1. A=(53591+J(K)·997) % Q
2. B=10267·(J(K)+1) % Q
3. Y=(B+X·A) % Q
4. v=Rand[Y, 0, $2^{20}$]
5. d=Deg[v]
6. a=1+Rand[Y, 1, L'−1]
7. b=Rand[Y, 2, L']

B.6 FEC Decoder Implementations

B.6.1 General

This section describes an efficient decoding algorithm for the MSCR codes described in this specification. Note that each received encoding symbol can be considered as the value of an equation amongst the intermediate symbols. From these simultaneous equations, and the known pre-coding relationships amongst the intermediate symbols, any algorithm for solving simultaneous equations can successfully decode the intermediate symbols and hence the source symbols. However, the algorithm chosen has a major effect on the computational efficiency of the decoding.

B.6.2 Decoding a Source Block

B.6.2.1 General

It is assumed that the decoder knows the structure of the source block it is to decode, including the symbol size, T, and the number K of symbols in the source block.

From the algorithms described in Sections B.5, the MSCR decoder can calculate the total number L=K+S+H of pre-coding symbols and determine how they were generated from the source block to be decoded. In this description it is assumed that the received encoding symbols for the source block to be decoded are passed to the decoder. Furthermore, for each such encoding symbol it is assumed that the number and set of intermediate symbols whose exclusive-or is equal to the encoding symbol is passed to the decoder. In the case of source symbols, the source symbol triples described in Section 8.5.2.2 indicate the number and set of intermediate symbols which sum to give each source symbol.

Let N≥K be the number of received encoding symbols for a source block and let M=S+H+N. The following M by L bit matrix A can be derived from the information passed to the decoder for the source block to be decoded. Let C be the column vector of the L intermediate symbols, and let D be the column vector of M symbols with values known to the receiver, where the first S+H of the M symbols are zero-valued symbols that correspond to LDPC and Half symbols (these are check symbols for the LDPC and Half symbols, and not the LDPC and Half symbols themselves), and the remaining N of the M symbols are the received encoding symbols for the source block. Then, A is the bit matrix that satisfies A·C=D, where here · denotes matrix multiplication over GF[2]. In particular, A[i, j]=1 if the intermediate symbol corresponding to index j is exclusive-ORed into the LDPC, Half or encoding symbol corresponding to index i in the encoding, or if index i corresponds to a LDPC or Half symbol and index j corresponds to the same LDPC or Half symbol. For all other i and j, A[i, j]=0.

Decoding a source block is equivalent to decoding C from known A and D. It is clear that C can be decoded if and only if the rank of A over GF[2] is L. Once C has been decoded, missing source symbols can be obtained by using the source symbol triples to determine the number and set of intermediate symbols which are exclusive-ORed to obtain each missing source symbol.

The first step in decoding C is to form a decoding schedule. In this step A is converted, using Gaussian elimination (using row operations and row and column reorderings) and after discarding M−L rows, into the L by L identity matrix. The decoding schedule comprises the sequence of row operations and row and column re-orderings during the Gaussian elimination process, and only depends on A and not on D. The decoding of C from D can take place concurrently with the forming of the decoding schedule, or the decoding can take place afterwards based on the decoding schedule.

The correspondence between the decoding schedule and the decoding of C is as follows. Let c[0]=0, c[1]=1 . . . , c[L−1]=L−1 and d[0]=0, d[1]=1 . . . , d[M−1]=M−1 initially.

Each time row i of A is exclusive-ORed into row i' in the decoding schedule then in the decoding process symbol D[d[i]] is exclusive-ORed into symbol D[d[i']].

Each time row i is exchanged with row i' in the decoding schedule then in the decoding process the value of d[i] is exchanged with the value of d[i'].

Each time column j is exchanged with column j' in the decoding schedule then in the decoding process the value of c[j] is exchanged with the value of c[j'].

From this correspondence it is clear that the total number of exclusive-ORs of symbols in the decoding of the source block is the number of row operations (not exchanges) in the Gaussian elimination. Since A is the L by L identity matrix after the Gaussian elimination and after discarding the last M−L rows, it is clear at the end of successful decoding that the L symbols D[d[0]], D[d[1]], . . . , D[d[L−1]] are the values of the L symbols C[c[0]], C[c[1]], . . . , C[c[L−1]].

The order in which Gaussian elimination is performed to form the decoding schedule has no bearing on whether or not the decoding is successful. However, the speed of the decoding depends heavily on the order in which Gaussian elimination is performed. (Furthermore, maintaining a sparse representation of A is crucial, although this is not described here). The remainder of this section describes an order in which Gaussian elimination could be performed that is relatively efficient.

B.6.2.2 First Phase

The first phase of the Gaussian elimination the matrix A is conceptually partitioned into submatrices. The submatrix sizes are parameterized by non-negative integers i and u which are initialized to 0. The submatrices of A are:

(1) The submatrix I defined by the intersection of the first i rows and first i columns. This is the identity matrix at the end of each step in the phase.

(2) The submatrix defined by the intersection of the first i rows and all but the first i columns and last u columns. All entries of this submatrix are zero.

(3) The submatrix defined by the intersection of the first i columns and all but the first i rows. All entries of this submatrix are zero.

(4) The submatrix U defined by the intersection of all the rows and the last u columns.

(5) The submatrix V formed by the intersection of all but the first i columns and the last u columns and all but the first i rows.

FIG. **22** illustrates the submatrices of A. At the beginning of the first phase V=A. In each step, a row of A is chosen. The following graph defined by the structure of V is used in determining which row of A is chosen. The columns that intersect V are the nodes in the graph, and the rows that have exactly 2 ones in V are the edges of the graph that connect the two columns (nodes) in the positions of the two ones. A component in this graph is a maximal set of nodes (columns) and edges (rows) such that there is a path between each pair of nodes/edges in the graph. The size of a component is the number of nodes (columns) in the component.

There are at most L steps in the first phase. The phase ends successfully when i+u=L, i.e., when V and the all zeroes submatrix above V have disappeared and A comprises I, the all zeroes submatrix below I, and U. The phase ends unsuccessfully in decoding failure if at some step before V disappears there is no non-zero row in V to choose in that step. In each step, a row of A is chosen as follows:

If all entries of V are zero then no row is chosen and decoding fails.

Let r be the minimum integer such that at least one row of A has exactly r ones in V.

If r≠2 then choose a row with exactly r ones in V with minimum original degree among all such rows.

If r=2 then choose any row with exactly 2 ones in V that is part of a maximum size component in the graph defined by X.

After the row is chosen in this step the first row of A that intersects V is exchanged with the chosen row so that the chosen row is the first row that intersects V. The columns of A among those that intersect V are reordered so that one of the r ones in the chosen row appears in the first column of V and so that the remaining r−1 ones appear in the last columns of V. Then, the chosen row is exclusive-ORed into all the other rows of A below the chosen row that have a one in the first column of V. Finally, i is incremented by 1 and u is incremented by r−1, which completes the step.

B.6.2.3 Second Phase

The submatrix U is further partitioned into the first i rows, $U_{upper}$, and the remaining M−i rows, $U_{lower}$. Gaussian elimination is performed in the second phase on $U_{lower}$ to either determine that its rank is less than u (decoding failure) or to convert it into a matrix where the first u rows is the identity matrix (success of the second phase). Call this u by u identity matrix $I_u$. The M−L rows of A that intersect $U_{lower}-I_u$ are discarded. After this phase A has L rows and L columns.

B.6.2.4 Third Phase

After the second phase the only portion of A which needs to be zeroed out to finish converting A into the L by L identity matrix is $U_{upper}$. The number of rows i of the submatrix $U_{upper}$ is generally much larger than the number of columns u of $U_{upper}$. To zero out $U_{upper}$ efficiently, the following precomputation matrix U' is computed based on $I_u$ in the third phase and then U' is used in the fourth phase to zero out $U_{upper}$. The u rows of $I_u$ are partitioned into ceil(u/8) groups of 8 rows each. Then, for each group of 8 rows all non-zero combinations of the 8 rows are computed, resulting in $2^8-1=255$ rows (this can be done with $2^8-8-1=247$ exclusive-ors of rows per group, since the combinations of Hamming weight one that appear in $I_u$ do not need to be recomputed). Thus, the resulting precomputation matrix U' has ceil(u/8)·255 rows and u columns. Note that U' is not formally a part of matrix A, but will be used in the fourth phase to zero out $U_{upper}$.

B.6.2.5 Fourth Phase

For each of the first i rows of A, for each group of 8 columns in the $U_{upper}$ submatrix of this row, if the set of 8 column entries in $U_{upper}$ are not all zero then the row of the precomputation matrix U' that matches the pattern in the 8 columns is exclusive-ORed into the row, thus zeroing out those 8 columns in the row at the cost of exclusive-oring one row of U' into the row.

After this phase A is the L by L identity matrix and a complete decoding schedule has been successfully formed. Then, the corresponding decoding comprising exclusive-ORing known encoding symbols can be executed to recover the intermediate symbols based on the decoding schedule.

The triples associated with all source symbols are computed according to B.5.2.2. The triples for received source symbols are used in the decoding. The triples for missing source symbols are used to determine which intermediate symbols need to be exclusive-ORed to recover the missing source symbols.

Some Properties of Some Multi-Stage Codes

In most of the examples described above, the input and output symbols encode **98** for the same number of bits and each output symbol is placed in one packet (a packet being a unit of transport that is either received in its entirety or lost in its entirety). In some embodiments, the communications system is modified so that each packet contains several output symbols. The size of an output symbol value is then set to a size determined by the size of the input symbol values in the initial splitting of the file or blocks of the stream into input symbols, based on a number of factors. The decoding process remains essentially unchanged, except that output symbols arrive in bunches as each packet is received.

The setting of input symbol and output symbol sizes is usually dictated by the size of the file or block of the stream and the communication system over which the output symbols are to be transmitted. For example, if a communication system groups bits of data into packets of a defined size or groups bits in other ways, the design of symbol sizes begins with the packet or grouping size. From there, a designer would determine how many output symbols will be carried in one packet or group and that determines the output symbol size. For simplicity, the designer would likely set the input symbol size equal to the output symbol size, but if the input data makes a different input symbol size more convenient, it can be used.

The above-described encoding process produces a stream of packets containing output symbols based on the original file or block of the stream. Each output symbol in the stream is generated independently of all other output symbols, and there is no lower or upper bound on the number of output symbols that can be created. A key is associated with each output symbol. That key, and some contents of the input file or block of the stream, determines the value of the output symbol. Consecutively generated output symbols need not have consecutive keys, and in some applications it would be preferable to randomly generate the sequence of keys, or pseudorandomly generate the sequence.

Multi-stage decoding has a property that if the original file or block of the stream can be split into K equal-sized input symbols and each output symbol value is the same length as an input symbol value, then the file or block can be recovered from K+A output symbols on average, with very high probability, where A is small compared to K. For example, for the weight distributions introduced above, the probability that the value of A exceeds $\alpha$*K is at most $10^{-12}$ if K is larger than 19,681, and it is at most $10^{-10}$ for any value of K. Since the particular output symbols are generated in a random or pseudorandom order, and the loss of particular output symbols in transit is assumed random, some small variance exists in the actual number of output symbols needed to recover the input file or block. In some cases, where a particular collection of K+A packets are not enough to decode the entire input file or block, the input file or block is still recoverable if the receiver can gather more packets from one or more sources of output packets.

Because the number of output symbols is only limited by the resolution of I, well more than K+A output symbols can be generated. For example, if I is a 32-bit number, 4 billion different output symbols could be generated, whereas the file or block of the stream could include K=50,000 input symbols. In some applications, only a small number of those 4 billion output symbols may be generated and transmitted and it is a near certainty that an input file or block of a stream can be recovered with a very small fraction of the possible output symbols and an excellent probability that the input file or block can be recovered with slightly more than K output symbols (assuming that the input symbol size is the same as the output symbol size).

In some applications, it may be acceptable to not be able to decode all of the input symbols, or to be able to decode all of input symbols, but with a relatively low probability. In such applications, a receiver can stop attempting to decode all of the input symbols after receiving K+A output symbols. Or, the receiver can stop receiving output symbols after receiving less than K+A output symbols. In some applications, the receiver may even only receive K or less output symbols. Thus, it is to be understood that in some embodiments of the present invention, the desired degree of accuracy need not be complete recovery of all the input symbols.

Further, in some applications where incomplete recovery is acceptable, the data can be encoded such that all of the input symbols cannot be recovered, or such that complete recovery of the input symbols would require reception of many more output symbols than the number of input symbols. Such an encoding would generally require less computational expense, and may thus be an acceptable way to decrease the computational expense of encoding.

It is to be understood that the various functional blocks in the above-described figures may be implemented by a combination of hardware and/or software, and that in specific implementations some or all of the functionality of some of the blocks may be combined. Similarly, it is also to be understood that the various methods described herein may be implemented by a combination of hardware and/or software.

The above description is illustrative and not restrictive. Many variations of the invention will become apparent to those of skill in the art upon review of this disclosure. The scope of the invention should, therefore, be determined not with reference to the above description, but instead should be determined with reference to the appended claims along with their full scope of equivalents.

What is claimed is:

**1**. A method of encoding data for transmission from a source to a destination over a communications channel, wherein the data for transmission is representable by an ordered set of input symbols, the method comprising:

generating a plurality of redundant symbols from the ordered set of input symbols, wherein each redundant symbol in the plurality of redundant symbols is calculated using one or more of the ordered set of input symbols; and

generating a plurality of output symbols from a combined set of symbols including the input symbols and the redundant symbols, wherein the number of possible output symbols is much larger than the number of symbols in the combined set of symbols, wherein at least one output symbol is generated from more than one symbol in the combined set of symbols and from less than all of the symbols in the combined set of symbols,

wherein the generating of the plurality of redundant symbols is done using a deterministic process that ensures low common membership among redundant symbols,

wherein common membership between two redundant symbols is the set of input symbols that both redundant symbols depend upon and wherein low common membership between two redundant symbols is present when the number of input symbols that both redundant symbols depend upon is less than a threshold as given by said process and

wherein for each input symbol there is a predetermined number of redundant symbols that depend upon said input symbol.

**2**. The method of claim **1**, further comprising transmitting the plurality of output symbols over said communications channel.

**3**. The method of claim **1**, further comprising storing the plurality of output symbols on a storage media.

**4**. The method of claim **1**, wherein the plurality of redundant symbols is generated according to a LDPC code.

**5**. The method of claim **1**, wherein output symbols are such that the ordered set of input symbols can be regenerated from any predetermined number, N, of the output symbols, where N is slightly larger than the number of input symbols.

**6**. The method of claim **1**, wherein the output symbols are such that there is a high probability that the ordered set of input symbols can be regenerated from N of the output symbols where N is at least as large as the number of input symbols.

**7**. The method of claim **1**, wherein the output symbols are such that G of the ordered set of input symbols can be regenerated from K of the output symbols where K is the number of input symbols and G is less than K.

**8**. The method of claim **1**, wherein at most G input symbols can be regenerated from any number of output symbols, wherein G is less than the number of input symbols in the ordered set of input symbols.

**9**. The method of claim **1**, wherein generating a plurality of redundant symbols includes, for each redundant symbol:

determining t distinct input symbols according to a weight distribution; and

computing each redundant symbol as the XOR of the t distinct input symbols.

**10**. The method of claim **1**, further comprising transmitting the plurality of output symbols over said communications channel, wherein the step of generating the plurality of output symbols is performed substantially concurrently with the step of transmitting the plurality of output symbols.

**11**. The method of claim **1**, wherein the plurality of redundant symbols comprises static symbols, Hamming symbols and padding symbols, wherein the sum of the number of symbols is selected to be a prime number.

**12**. The method of claim **1**, wherein the predefined threshold is six, such that for any two distinct redundant symbols, the sets of input symbols that each of those two distinct redundant symbols depend on have at most six input symbols in common.

**13**. The method of claim **1**, wherein generating the plurality of redundant symbols using one or more of the ordered set of input symbols comprises:

initializing an array C[K], . . . , C[K+S−1] so that each array element is a known value, wherein K is the number of input symbols and S is the number of redundant symbols to be generated and C[0], . . . , C[K−1] corresponds to the K input symbols;

performing the following steps, with a counter i=0, wherein a and b are intermediate variables, "%" represents a modulo operation, floor( ) is a function representing a highest integer value that is less than the function's argument, and "ˆ" represents a bitwise XOR operation:

(1) a=1+(floor(i/S) % (S−1))
(2) b=1% S
(3) C[K+b]=C[K+b]ˆC[i]
(4) b=(b+a) % S
(5) C[K+b]=C[K+b]ˆC[i]
(6) b=(b+a) % S
(7) C[K+b]=C[K+b]ˆC[i]

repeating those steps, for each value of counter i from 1 to K−1; and

outputting at least the resulting array C[K], . . . , C[K+S−1] as the plurality of S redundant symbols.

**14**. The method of claim **13**, wherein S is the smallest prime integer such that S≥ceil(0.01·K)+X, where X is the smallest positive integer such that X·(X−1)=2·K.

**15**. The method of claim **1**, wherein for each input symbol the predetermined number of redundant symbols that depend upon said input symbol is three.

**16**. A system for encoding data for transmission from a source to a destination over a communications channel, wherein the data for transmission is representable by an ordered set of input symbols, the system comprising:

a static encoder configured to generate a plurality of redundant symbols from the ordered set of input symbols, wherein each redundant symbol in the plurality of redundant symbols is calculated using one or more of the ordered set of input symbols; and

a dynamic encoder communicatively coupled to the static encoder and configured to generate a plurality of output symbols from a combined set of symbols including the input symbols and the redundant symbols, wherein the number of possible output symbols is much larger than the number of symbols in the combined set of symbols, wherein at least one output symbol is generated from more than one symbol in the combined set of symbols and from less than all of the symbols in the combined set of symbols,

wherein the static encoder is configured to generate the plurality of redundant symbols using a deterministic process that ensures low common membership among redundant symbols,

wherein common membership between two redundant symbols is the set of input symbols that both redundant symbols depend upon and wherein low common membership between two redundant symbols is present when the number of input symbols that both redundant symbols depend upon is less than a threshold as given by said process and

wherein for each input symbol there is a predetermined number of redundant symbols that depend upon said input symbol.

**17**. The system of claim **16**, further comprising a transmit module communicatively coupled to the dynamic encoder and configured to transmit the plurality of output symbols over said communications channel.

**18**. The system of claim **16**, wherein the static encoder is configured to generate the plurality of redundant symbols using one or more of the ordered set of input symbols including:

initializing an array C[K], . . . , C[K+S−1] so that each array element is a known value, wherein K is the number of input symbols and S is the number of redundant symbols to be generated and C[0], . . . , C[K−1] corresponds to the K input symbols;

performing the following steps, with a counter i=0, wherein a and b are intermediate variables, "%" represents a modulo operation, floor( ) is a function representing a highest integer value that is less than the function's argument, and "ˆ" represents a bitwise XOR operation:

(1) a=1+(floor(i/S) % (S−1))
(2) b=1% S
(3) C[K+b]=C[K+b]ˆC[i]
(4) b=(b+a) % S
(5) C[K+b]=C[K+b]ˆC[i]
(6) b=(b+a) % S
(7) C[K+b]=C[K+b]ˆC[i]

repeating those steps, for each value of counter i from 1 to K−1; and

outputting at least the resulting array C[K], . . . , C[K+S−1] as the plurality of S redundant symbols.

**19**. The system of claim **18**, wherein S is the smallest prime integer such that S≥ceil(0.01·K)+X, where X is the smallest positive integer such that X·(X−1)=2·K.

**20**. A non-transitory computer-readable medium for use with electronics capable of executing instructions read from the computer-readable medium in order to implement encoding data for transmission from a source to a destination over a communications channel, wherein the data for transmission is representable by an ordered set of input symbols, the computer-readable medium having stored thereon:

program code for generating a plurality of redundant symbols from the ordered set of input symbols, wherein each redundant symbol in the plurality of redundant symbols is calculated using one or more of the ordered set of input symbols; and

program code for generating a plurality of output symbols from a combined set of symbols including the input symbols and the redundant symbols, wherein the number of possible output symbols is much larger than the number of symbols in the combined set of symbols, wherein at least one output symbol is generated from more than one symbol in the combined set of symbols and from less than all of the symbols in the combined set of symbols,

wherein the program code for generating the plurality of redundant symbols uses a deterministic process that ensures low common membership among redundant symbols,

wherein common membership between two redundant symbols is the set of input symbols that both redundant symbols depend upon and wherein low common membership between two redundant symbols is present when the number of input symbols that both redundant symbols depend upon is less than a threshold as given by said process and

wherein for each input symbol there is a predetermined number of redundant symbols that depend upon said input symbol.

**21**. A system for encoding data for transmission from a source to a destination over a communications channel, wherein the data for transmission is representable by an ordered set of input symbols, the system comprising:

means for generating a plurality of redundant symbols from the ordered set of input symbols, wherein each redundant symbol in the plurality of redundant symbols is calculated using one or more of the ordered set of input symbols; and

means for generating a plurality of output symbols from a combined set of symbols including the input symbols and the redundant symbols, wherein the number of possible output symbols is much larger than the number of symbols in the combined set of symbols, wherein at least one output symbol is generated from more than one symbol in the combined set of symbols and from less than all of the symbols in the combined set of symbols,

wherein the means for generating the plurality of redundant symbols includes means for using a deterministic process that ensures low common membership among redundant symbols,

wherein common membership between two redundant symbols is the set of input symbols that both redundant symbols depend upon and wherein low common membership between two redundant symbols is present when the number of input symbols that both redundant symbols depend upon is less than a threshold as given by said process and

wherein for each input symbol there is a predetermined number of redundant symbols that depend upon said input symbol.

**22**. A system for decoding encoded data received over a communications channel transmitted from a source to a destination, the system comprising:

a receive module configured to receive a predetermined number, N, of symbols, wherein the received symbols comprise a combination of received source symbols and received repair symbols generated from a plurality of an ordered set of K source symbols; and

a decoder communicatively coupled to the receive module and configured to generate to a desired degree of accuracy one or more unreceived source symbols of the ordered set of K source symbols,

wherein each received symbol has an associated symbol relation that is determined by a systematic index, J(K), where J(K) is determined by K,

wherein the value of each unreceived source symbol is determined by the associated symbol relation and a plurality of L intermediate symbol values, wherein L is at least K,

wherein the L intermediate symbol values are determined by the K source symbol values and by the K symbol relations associated with the K source symbols and by a set of L-K pre-coding relations, and

wherein the L intermediate symbol values can be generated to a desired degree of accuracy from the N received source and repair symbols.

**23**. The system of claim **22**, wherein each source symbol has an associated encoding symbol identifier ("ESI") that identifies the source symbol, wherein the systematic index J(K) and a value X, wherein X is a valid ESI, determines the symbol relation for the source symbol identified by ESI X.

**24**. The system of claim **22**, wherein the number L-K of pre-coding relations comprises a first set of S pre-coding relations and a second set of H pre-coding relations, and wherein the L intermediate symbols comprises a first set of K intermediate symbols, a second set of S intermediate symbols, and a third set of H intermediate symbols.

**25**. The system of claim **22**, wherein the K source symbols correspond to a source block, wherein the source block is defined by a transport protocol for streaming data.

**26**. A non-transitory computer-readable medium for use with electronics capable of executing instructions read from the computer-readable medium in order to implement decoding encoded data received over a communications channel transmitted from a source to a destination, the computer-readable medium having stored thereon:

program code for receiving a predetermined number, N, of symbols, wherein the received symbols comprise a combination of received source symbols and received repair symbols generated from a plurality of an ordered set of K source symbols; and

program code for generating to a desired degree of accuracy one or more unreceived source symbols of the ordered set of K source symbols,

wherein each received symbol has an associated symbol relation that is determined by a systematic index, J(K), where J(K) is determined by K,

wherein the value of each unreceived source symbol is determined by the associated symbol relation and a plurality of L intermediate symbol values, wherein L is at least K,

wherein the L intermediate symbol values are determined by the K source symbol values and by the K symbol relations associated with the K source symbols and by a set of L-K pre-coding relations, and

wherein the L intermediate symbol values can be generated to a desired degree of accuracy from the N received source and repair symbols.

**27**. A system for decoding encoded data received over a communications channel transmitted from a source to a destination, the system comprising:

means for receiving a predetermined number, N, of symbols, wherein the received symbols comprise a combination of received source symbols and received repair symbols generated from a plurality of an ordered set of K source symbols; and

means for generating to a desired degree of accuracy one or more unreceived source symbols of the ordered set of K source symbols,

wherein each received symbol has an associated symbol relation that is determined by a systematic index, J(K), where J(K) is determined by K,

wherein the value of each unreceived source symbol is determined by the associated symbol relation and a plurality of L intermediate symbol values, wherein L is at least K,

wherein the L intermediate symbol values are determined by the K source symbol values and by the K symbol relations associated with the K source symbols and by a set of L-K pre-coding relations, and

wherein the L intermediate symbol values can be generated to a desired degree of accuracy from the N received source and repair symbols.

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